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REGGE DESCRIPTION OF SPIN-SPIN ASYMMETRY IN PHOTON DIFFRACTIVE DISSOCIATION

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Abstract

We explore the possibility whether the gluon helicity distribution $\Delta G(x)$ can be extracted from a comparison of experimental data on the longitudinal spin-spin asymmetry A_{LL} in γp diffractive deep inelastic scattering with calculations performed within the framework of perturbative QCD (pQCD). The data could be obtained at the future HERA collider in scattering of polarized electrons/positrons off polarized protons. In this paper we look for such kinematical regions where contributions to A_{LL} from soft processes (reggeon exchanges) are suppressed to guarantee an applicability of pQCD. It is shown that for the square of the center-of-mass energy $s_{\gamma p} \geq 10^3$ GeV^2, the hadronic diffractive mass $M_X \leq 10$ GeV/c^2, the momentum transferred to the proton $\Delta_T \leq 0.5$ GeV/c, and $Q^2 \geq 4$ (GeV/c)^2 the longitudinal spin-spin asymmetry due to reggeon exchanges is less than 10^{-4}. This value is presumably lower than the asymmetry which can be measured with modern experimental technique. This means that the pQCD prediction can be reliably compared with data in this kinematical region.

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1 Introduction

Study of double spin asymmetries in deep inelastic scattering (DIS) of leptons on nucleons is of great importance as it provides valuable information about spin-dependent parton distributions in the nucleon. An extraction of the gluon helicity distribution from DIS is the most difficult procedure and gives large uncertainties as the gluon is an electrically neutral particle. Recently it has been argued that the diffractive ρ-meson, open charm electroproduction [1], [2], [3], [4] and the production of di-jets [5], [6], [7], [8] in hard collisions of polarized electrons with protons can be used for the investigation of the unpolarized gluon density and the gluon helicity distribution in the proton. Such predictions have been made in the framework of perturbative QCD when typical transverse momenta of produced particles and total created masses are large enough for perturbative QCD to be applicable. To decrease statistical errors we have to consider events with hadron transverse momenta and total created masses of the order of 1 GeV where correction of soft processes can be appreciable. So we need an estimate of the corrections to perturbative QCD predictions.

The physical picture of photon diffractive dissociation is as follows. The virtual photon produced by a scattered lepton dissociates into a quark-antiquark pair which gives final hadrons after rescattering on the proton. In perturbative QCD, scattering of the $q\bar{q}$-pair off the proton is described by the gluon ladder graphs which correspond to the hard part of pomeron exchange. To estimate the soft part contribution to the amplitude of $q\bar{q}$-pair scattering off the proton we shall make use of the Regge phenomenological approach. In the sixties and seventies the Regge complex angular momentum theory has been very successfully applied to the description of elastic scattering and charge exchange reactions for different hadrons (see, for example, reviews [9], [10]). We shall apply the parameters of the Regge trajectories and the hadron-reggeon vertices presented in [10], [11], [12] which have been found from the phenomenological analysis of experimental data on hadron-hadron collisions at high energies. To extract quark-reggeon vertices we make use of the nucleon wave function in the naive quark model [13]. The obtained quark-reggeon vertex parameters depend of course on the applied model of the nucleon but we hope that for our rough estimates of the order of magnitude of the soft part contribution to the longitudinal spin-spin asymmetry this approach gives reasonable results. It is true mainly due to the fact that the main aim of the present paper consists in finding such kinematical conditions where the soft process contribution to the asymmetry is much less than the prediction of perturbative QCD and hence we do not need the real theoretical calculation of the Regge pole contribution but some estimate only.

In the Regge phenomenology all invariant amplitudes of quark proton scattering due to exchange with a reggeon R contain the factor $(s/s_0)^{\alpha_R(0)-1}$ where s is the square of the total center-of-mass energy of the colliding particles, $s_0 \sim 1$ GeV^2 denotes the parameter in the Regge theory and $\alpha_R(t)$ is a Regge trajectory depending on t (the square of the reggeon momentum). For the vacuum (pomeron) trajectory $\alpha_R(0) \approx 1$, for f, ρ, ω, A_2 reggeons $\alpha_R(0) \approx 0.5$ and for the π and $A_1(1260)$ trajectories $\alpha_R(0) \leq 0$. We see that there is the hierarhy of reggeon contributions valid at large s. Since both for quark and antiquark scattering on the proton the square of the total center-of-mass energy is proportional to $s_{\gamma p}$ (the Mandelstam variable for the γp-collision) it is convenient

to introduce the small parameter $\epsilon = \sqrt{s_0/s_{\gamma p}}$ for the classification of reggeon exchange contributions. We decompose the cross section and the longitudinal spin-spin asymmetry, A_{LL} into a power series in ϵ and study properties of terms $\sim \epsilon^0$, ϵ^1 and ϵ^2.

We apply our formulæ to γp-scattering at energies achieved at the HERA collider. We have found that though the pure pomeron contribution to A_{LL} is a quantity $\sim \epsilon^0$ it is numerically much smaller than the contributions $\sim \epsilon^1$ and ϵ^2 at $s_{\gamma p} = 10^2$ to 10^5 GeV^2 and for the zero momentum transfer. More over the contributions $\sim \epsilon^1$ to A_{LL} representing the interference terms between pomeron and $f,\ \rho,\ \omega,\ A_2$ exchanges are numerically much less than the terms $\sim \epsilon^2$ at the HERA energies. Here we suppose the pomeron contribution to be a sum of the pole term and the cuts in the complex plane of the angular momentum due to exchanges with the n pomerons ($n \geq 2$). The $f,\ \rho,\ \omega,\ A_2$ contributions are also assumed to be sums of $f,\ \rho,\ \omega,\ A_2$ exchanges and some number of pomeron exchanges. The explanation of this numerical hierarhy being inverse to the parameteric hierarhy $\epsilon^0 \gg \epsilon^1 \gg \epsilon^2$ is as follows. The fit [12], [14] of experimental data has shown that the spin-dependent parts of the quark-quark-pomeron and nucleon-nucleon-pomeron vertices are small compared with the scalar parts. Besides the pomeron contribution to the numerator in the formula for A_{LL} at $t = 0$ contains the sum of the squares of the invariant amplitudes which vanish in the one and two pomeron exchange approximations. The nonzero invariant amplitudes contain the third or higher powers of the small spin-dependent pomeron vertices and this leads to the very small ($\leq 10^{-12}$) contribution to A_{LL}. For nonzero momentum transfers (at $\Delta_T \sim 1$ GeV/c) $|A_{LL}|$ is greater by few orders of magnitude than at $t = 0$. The contributions to A_{LL} $\sim \epsilon^1$ and $\sim \epsilon^2$ became numerically comparable with each other at $|t| \sim 1$ (GeV/c)^2 but the contribution $\sim \epsilon^0$ is much smaller than the first and second order contributions to A_{LL}.

Formally the amplitudes of π and $A_1(1260)$ exchanges are quantities $\sim \epsilon^2$ and are to be smaller than the amplitudes of $f,\ \rho,\ \omega,\ A_2$ exchanges which are $\sim \epsilon^1$. This is true for $s_{\gamma p} \to \infty$ but it is wrong numerically at the HERA energies. Indeed, we have for the square of the quark-proton center-of-mass energy the relation $s = z s_{\gamma p}$ where z is the fraction of the virtual photon momentum carried by the quark (in the light front system of frame). It turns out that the dominant contribution of quark-proton scattering to A_{LL} originates from the region in which z is close to its minimal value $z_{min} \approx (m_q^2 + k_T^2)/M_X^2$. We denote by M_X the mass of hadrons produced in hard γp-scattering, k_T and m_q are the transverse momentum and the mass of the quark, respectively. For experiments at HERA z_{min} can be $\sim 10^{-2}$ - 10^{-3}, hence s can be ~ 1 GeV^2 even for $s_{\gamma p} \sim 10^2$ - 10^3 GeV^2. It is clear that the contributions of π, $A_1(1260)$ reggeons to A_{LL} at $s \sim 1$ GeV^2 can be of the same order of magnitude as the $f,\ \rho,\ \omega,\ A_2$ contributions. Our numerical calculations show that we can neglect the $A_1(1260)$ contribution to A_{LL} but the π-reggeon gives the appreciable contribution and sometimes the dominant contribution to A_{LL} even at $s_{\gamma p} \geq 10^3$ GeV^2 due to the large value of the πNN coupling constant.

Considering, for example, experimental events with relatively low M_X (say, $M_X \leq 3$ GeV/c^2) or large k_T ($k_T \geq 1$ GeV/c) we increase a value of z_{min} and hence we increase the minimal center-of-mass energy of quark-nucleon scattering (and antiquark-nucleon scattering too). As a result we reduce the contributions of secondary reggeons both with the natural parity ($f,\ \rho,\ \omega,\ A_2$) and with the unnatural parity (π, A_1). Their

contribution to A_{LL} can be suppressed down to 10^{-4}. This value is presumably the low limit for the longitudinal spin-spin asymmetry which can be measured by modern experimental techniques. The perturbative QCD contribution to A_{LL} can be calculated most unambiguously from the theoretical point of view. If it is larger than 10^{-4}, then the perturbative QCD prediction can be reliably compared with an experimental value of the longitudinal spin-spin asymmetry which can be obtained at the future HERA collider in scattering of the polarized electrons/positrons off the polarized protons.

The paper is organized as follows. In Sections 2 and 3 we consider spectator graphs only and discuss contributions to the longitudinal spin-spin asymmetry of pure pomeron exchanges and contributions of secondary Regge trajectories, respectively. Properties of nonspectator graphs are considered in Section 4. Results of numerical calculations of the asymmetry and discussion of cuts suppressing its value are presented in Section 5. Main results are summarized in the Conclusion. The most complicated formulæ applied for the numerical calculations are given in the Appendix.

2 Spectator diagrams. Pomeron contribution

We consider in the present paper unenhanced Regge diagrams only and divide the pomeron exchange graphs into two sets. The first type diagrams, called spectator graphs, are shown in Fig. 1a and Fig. 1b. They describe scattering of a quark (or antiquark) off the nucleon, the other particle of the $q\bar{q}$-pair being a spectator. The graphs of the second type are presented in Figs. 2a and 2b. They describe scattering both of the quark and antiquark on the nucleon and are named nonspectator graphs.

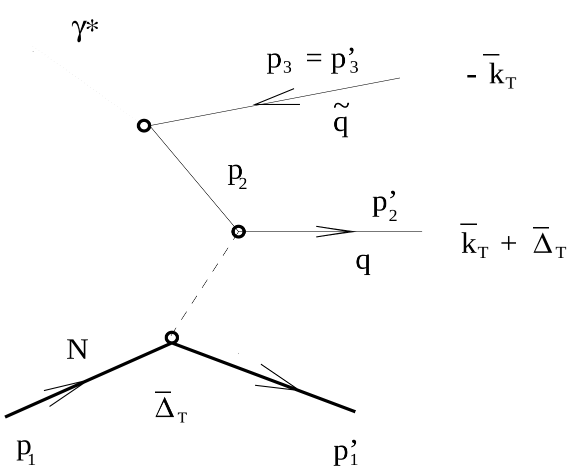


Fig. 1a: One reggeon exchange spectator graph. Dotted, thick straight and dashed lines correspond to photon, nucleon and reggeon, respectively. Straight lines describe either quarks or antiquarks.

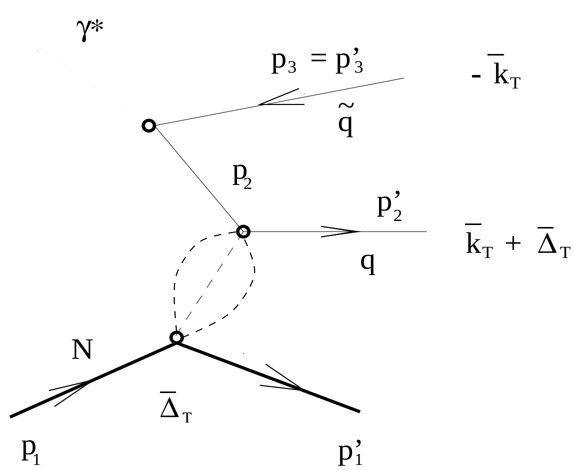


Fig. 1b: Spectator diagram with three reggeon exchanges. Lines have the same meaning as in Fig. 1a.

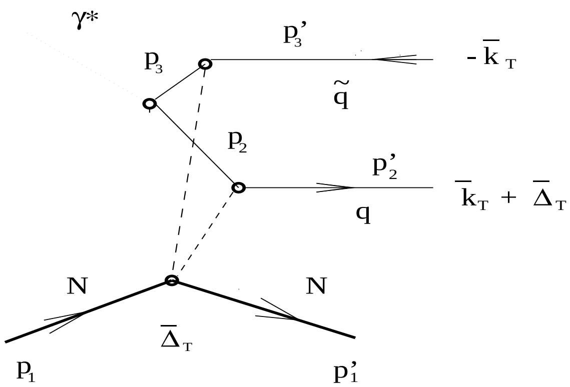


Fig. 2a: Non-spectator graph with two reggeon exchanges. Lines have the same meaning as in Fig. 1a.

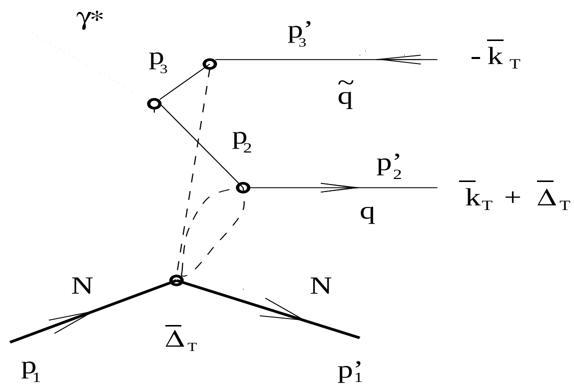


Fig. 2b: Non-spectator graph with three reggeon exchanges. Lines have the same meaning as in Fig. 1a.

The graphs presented in Figs. 1a and 1b describe the one and three pomeron exchanges, respectively. We restrict ourselves to graphs containing not more than three reggeon exchanges. In Fig. 1a $p_1,\ p_2,\ p_3$ are the incident momenta of the nucleon, quark, antiquark, respectively. Here and after we denote the nucleon as particle number 1, the quark and antiquark will be quoted as particles with numbers 2 and 3, respectively. The momenta of outgoing particles are denoted by $p'_1,\ p'_2,\ p'_3$, transferred four-momentum Δ is equal to $p'_1 - p_1$. We can present the amplitude of quark-nucleon scattering as a sum of the amplitudes of one, two, ..., n pomeron exchanges

$$\hat{A}_P = \hat{A}_P^{(1)} + \hat{A}_P^{(2)} + \hat{A}_P^{(3)} + ... \ . \tag{1}$$

The first term in (1) corresponds to the Regge pole contribution

$$\hat{A}_P^{(1)} = p(\Delta)P(\Delta)G_P(\Delta, s)\ , \tag{2}$$

where s is the square of the center-of-mass energy of the colliding particles, $G_P(\Delta, s)$ denotes the pomeron propagator, and $p(\Delta)$, and $P(\Delta)$ are quark-quark-pomeron (qqP) and nucleon-nucleon-pomeron (NNP) vertices, respectively. The formula for the NNP-vertex $P(\Delta)$ in the helicity representation reads

$$P_{\lambda_1}^{\lambda'_1}(\Delta) = (I_1)_{\lambda_1}^{\lambda'_1} P_s(\Delta^2) + iP_y(\Delta^2)(\vec{\sigma}_1 \cdot \vec{l} \times \vec{\Delta}_T)_{\lambda_1}^{\lambda'_1}\ , \tag{3}$$

where $\vec{l}$ denotes a unit vector along the quark three-momentum, λ_1 and λ'_1 are helicities of the initial and scattered nucleon, respectively (at high energy we neglect the masses of all particles, hence the nucleon helicity becomes a good quantum number.). The matrices I_1 and $\vec{\sigma}_1$ are the unit matrix and the Pauli matrices acting on the helicity variables of the nucleon. The functions $P_s(\Delta^2)$ and $P_y(\Delta^2)$ can be taken in the gaussian form [14]

$$P_s(\Delta^2) = P_s \exp\{-r_s^2 \Delta_T^2/2\}\ , \quad P_y(\Delta^2) = P_y \exp\{-r_y^2 \Delta_T^2/2\}\ , \tag{4}$$

where $\Delta_T = |\vec{\Delta}_T|$ and the three-vector $\vec{\Delta}_T$ denotes a transverse part of $\vec{\Delta}$ (orthogonal to the four-vectors p_1 and p_2) and $\Delta^2 \approx -\vec{\Delta}_T^2$. Parameterization (4) with $r_s = r_y \equiv r_P$

permitted to achieve in [11], [12] a reasonable description of the differential cross sections and polarizations in hadron-hadron collisions at beam energies 10 to 100 GeV. The values of the parameters P_s, P_y, r_P and parameters for ρ, f, A_2, ω reggeons are taken from [11], [12] and presented in Table 1.

Table 1.

Reggeon, a	P	ρ	f	A_2	ω
$\alpha_a(0)$	1.075	0.49	0.45	0.35	0.425
$\alpha'_a(0)$, GeV^{-2}	0.26	0.7	1.0	0.7	1.0
σ_a	+1	-1	+1	+1	-1
T_a	0	1	0	1	0
A_s, GeV^{-1}	2.28	0.41	2.5	0.41	1.94
A_y, GeV^{-2}	0.22	-1.52	0.32	-1.21	1.03
a_s, GeV^{-1}	0.76	0.41	0.83	0.41	0.65
a_y, GeV^{-2}	0.22	-0.91	0.32	-0.73	1.03
r_a^2, GeV^{-2}	2.3	3.46	5.2	2.0	9.1

We consider quarks and antiquarks as point-like particles so the quark-quark-pomeron (qqP) vertex $p(\Delta)$ looks like

$$p(\Delta) = p_s + ip_y(\vec{\sigma}_2 \cdot \vec{l} \times \vec{\Delta}_T) \ , \tag{5}$$

where p_s and p_y are some constants. To extract values of p_s and p_y from the NNP-vertex we consider the nucleon as a three quark system described by the nonrelativistic quark model. Applying the well known spin part of the proton wave functions [13]

$$\begin{gathered}|p_{\pm\frac{1}{2}}>= \frac{1}{\sqrt{18}}\Big\{\pm 2|u_{\pm\frac{1}{2}}u_{\pm\frac{1}{2}}d_{\mp\frac{1}{2}}> \pm 2|u_{\pm\frac{1}{2}}d_{\mp\frac{1}{2}}u_{\pm\frac{1}{2}}> \\ \pm 2|d_{\mp\frac{1}{2}}u_{\pm\frac{1}{2}}u_{\pm\frac{1}{2}}> \mp|u_{\pm\frac{1}{2}}u_{\mp\frac{1}{2}}d_{\pm\frac{1}{2}}> \mp|u_{\pm\frac{1}{2}}d_{\pm\frac{1}{2}}u_{\mp\frac{1}{2}}> \\ \mp|u_{\mp\frac{1}{2}}u_{\pm\frac{1}{2}}d_{\pm\frac{1}{2}}> \mp|d_{\pm\frac{1}{2}}u_{\pm\frac{1}{2}}u_{\mp\frac{1}{2}}> \mp|u_{\mp\frac{1}{2}}d_{\pm\frac{1}{2}}u_{\pm\frac{1}{2}}> \mp|d_{\pm\frac{1}{2}}u_{\mp\frac{1}{2}}u_{\pm\frac{1}{2}}>\Big\}\end{gathered} \tag{6}$$

one can get the relations of interest

$$p_s = \frac{1}{3}P_s \ , \quad p_y = P_y \ . \tag{7}$$

In (6) $|p_{\frac{1}{2}}>$ ($|p_{-\frac{1}{2}}>$) describes the proton with its spin parallel (antiparallel) to a quantization axis. The analogous meaning have quantities $|u_{\pm\frac{1}{2}}>$ and $|d_{\pm\frac{1}{2}}>$ for u- and d-quarks.

The propagator $G_a(\Delta, s)$ ($a = P,\ \rho,\ f,\ A_2,\ \omega$) describes exchange of a reggeon a with spin $\alpha_a(t)$ whose value depends on the square of the reggeon momentum ($t = \Delta^2 \approx -\vec{\Delta}_T^2$). The formulæ for $G_a(\Delta, s)$ reads [14]

$$G_a(\Delta, s) = \eta_a(t)(s/s_0)^{\alpha_a(t)-1} \ , \tag{8}$$

where s is the square of the center-of-mass energy. The parameter s_0 in the Regge theory for proton-proton scattering is put usually equal to $s_0 = 2m_pE_0$ with $E_0 = 1$

GeV where m_p is the proton mass. A word of caution is in order. In a more accurate consideration the quantity $\ln(s/s_0)$ should be replaced with the difference of the incident particle rapidity with respect to the target and some standard rapidity corresponding to s_0. Neglecting the motion and interaction of quarks in the proton we have in the naive quark model for quark-proton scattering at high energies the relation $s = s_{pp}/3$. Here s_{pp} denotes the square of the total energy in the center of mass of two protons. To have the same rapidity difference for the quark-proton system as for the proton-proton system we are to put $s_0 = 2m_pE_0/3$ in (8) for quark-proton scattering. The signature factor $\eta_a(t)$ is given by the relation [9], [14]

$$\eta_a(t) = -\frac{1 + \sigma_a \exp\{-i\pi\alpha_a(t)\}}{\sin \pi\alpha_a(t)} \ , \tag{9}$$

where σ_a is a signature of the Regge trajectory. For the trajectory on which the real particle (resonance) with spin equal to J lies, $\sigma_a = (-1)^J$, for the pomeron trajectory having the vacuum quantum numbers (vacuum trajectory) $\sigma_P = 1$. It is well known that at small t Regge trajectories $\alpha_a(t)$ are straight lines. Then putting the expression $\alpha_a(t) = \alpha_a(0) + \alpha_a'(0)t$ into (9) we get the approximate formula

$$\eta_a(t) = \eta_a(0) \exp\{t\frac{\pi}{2}\sigma_a\alpha_a'(0)\eta_a^*(0)\} \ , \tag{10}$$

where η_a^* denotes the complex conjugate quantity. Since for the linear trajectory we have

$$(s/s_0)^{\alpha_a(t)} = (s/s_0)^{\alpha_a(0)} \exp\{t\alpha_a'(0)\ln(s/s_0)\} \ , \tag{11}$$

we get instead of (8), taking into account (10), (11), the relation

$$G_a(\Delta, s) \approx \eta_a(0)(s/s_0)^{\alpha_a(0)-1} \exp\{-\alpha_a'(0)[\ln(s/s_0) + \frac{\pi}{2}\sigma_a\eta_a(0)^*]\Delta_T^2\} \tag{12}$$

which is convenient for numerical calculations.

The contributions of n pomeron exchange corresponding to the graph presented in Fig. 1b in accordance with the Gribov diagram technique are [15], [16]

$$A_P^{(n)} = \frac{i^{n-1}}{\pi^{n-1}n!} \int N_1^n N_2^n G_P(\vec{\Delta}_1, s)G_P(\vec{\Delta}_2, s)...G_P(\vec{\Delta}_n, s)$$
$$\delta(\sum_{i=1}^{n}\vec{\Delta}_i - \vec{\Delta})d^2\vec{\Delta}_1 d^2\vec{\Delta}_2...d^2\vec{\Delta}_n \ , \tag{13}$$

where $N_j^n \equiv N_j^n(\vec{\Delta}_1, \vec{\Delta}_2, ..., \vec{\Delta}_n)$ denotes the vertex for emission of n pomerons by the jth particle ($j = 1,\ 2,\ 3$) and $\vec{\Delta}_n$ is a two dimensional vector orthogonal to the three momenta of the colliding particles in the center-of-mass. In the eikonal approximation the formula for the vertex N_1^n reads

$$N_1^n(\vec{\Delta}_1, \vec{\Delta}_2, ..., \vec{\Delta}_n) = C_{sh}^{(n)} P(\vec{\Delta}_1)P(\vec{\Delta}_2)...P(\vec{\Delta}_n) \ . \tag{14}$$

The graph corresponding to relation (14) with $C_{sh}^{(n)} = 1$ is shown in Fig. 3a. To take into account the possibility to produce a shower of particles after each rescattering of

the nucleon (such a process is shown in Fig. 3b) one can put [14] $C_{sh}^{(1)} = 1$ and for $n \geq 2$ $C_{sh}^{(n)} = C_{sh}^{(2)}(C_0)^{n-2}$ with $C_{sh}^{(2)} = \sqrt{1 + \sigma^{in}/\sigma^{el}}$ and C_0 being a free parameter. These parameters in our calculations have been put equal to $C_{sh}^{(2)} = \sqrt{1.3}$, $C_0 = \sqrt{1.57}$ in accordance with [11], [12] where they have been found from fitting experimental data. We denote by σ^{el} and σ^{in} the total cross sections of elastic and inelastic nucleon-nucleon scattering. For the quark-quark-reggeon vertex we put $C_{sh}^{(n)} = 1$ ignoring the possibility of shower production in soft scattering of the point-like quarks and antiquarks.

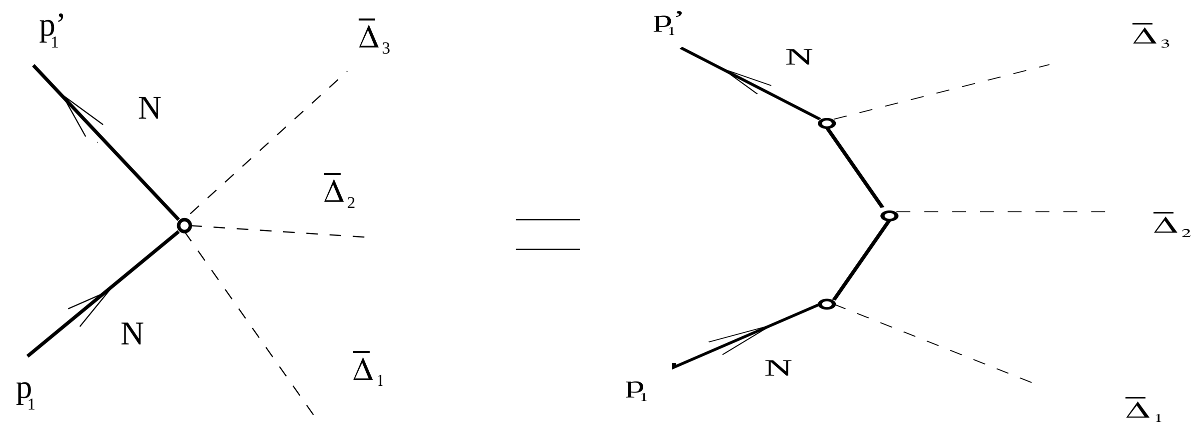


Fig. 3a: Nucleon-reggeon vertex for emission of three reggeons in eikonal approximation. Lines have the same meaning as in Fig. 1a.

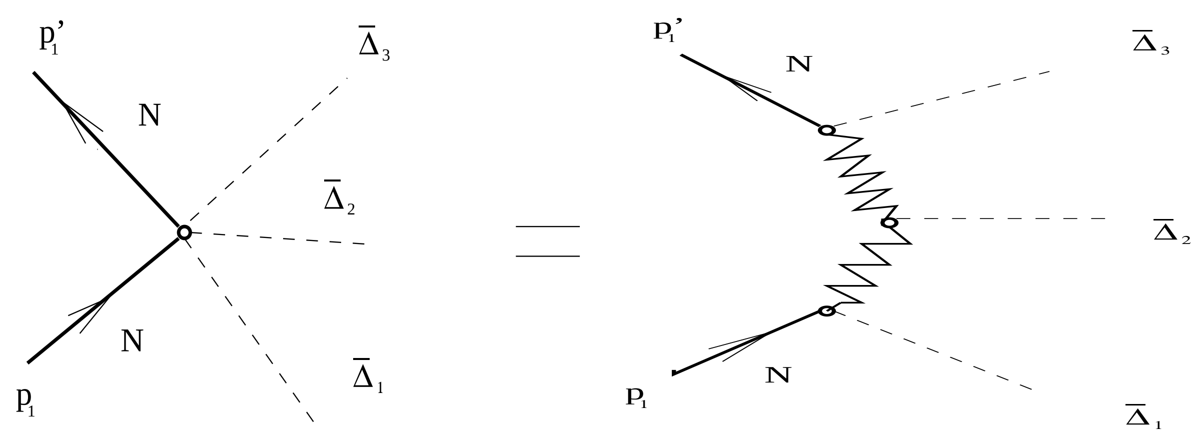


Fig. 3b: Nucleon-reggeon vertex for emission of three reggeons. Zigzags correspond to showers of intermediate particles between emissions of reggeons. Other lines have the same meaning as in Fig. 1a.

Formula (14) has to be corrected when $P(\vec{\Delta})$ depends on spin variables and hence $P(\vec{\Delta}_i)$ and $P(\vec{\Delta}_j)$ do not commute. The vertex $N_1^{(n)}$ should be symmetrized under all the permutations of momenta of reggeons $\vec{\Delta}_1,\ \vec{\Delta}_2, ...,\ \vec{\Delta}_n$ [17]. We would like to stress that we discuss the case of bosonic Regge trajectories. For pure pomeron exchanges this is obvious as pomerons are identical bosons. Hence we can write for the NNP-vertex

$$N_1^n(\vec{\Delta}_1, \vec{\Delta}_2, ..., \vec{\Delta}_n) = C_{sh}^{(n)}\{P(\vec{\Delta}_1)P(\vec{\Delta}_2)...P(\vec{\Delta}_n)\}$$
$$\equiv \frac{C_{sh}^{(n)}}{n!}\sum P(\vec{\Delta}_1)P(\vec{\Delta}_2)...P(\vec{\Delta}_n)\ , \tag{15}$$

where the brackets $\{\}$ in (15) mean a sum over all permutations of the momenta divided by $n!$ The symmetry property should be valid for the qqP-vertex, hence we write

$$N_j^n(\vec{\Delta}_1, \vec{\Delta}_2, ..., \vec{\Delta}_n) = \{p(\vec{\Delta}_1)p(\vec{\Delta}_2)...p(\vec{\Delta}_n)\} \tag{16}$$

with $j = 2,\ 3$ (recall that $j = 2$ and $j = 3$ correspond to the quark and antiquark, respectively). The general expression for the amplitude of elastic quark-nucleon scattering reads

$$\hat{A}(\vec{\Delta}_T) = A_1 + A_2(\vec{\sigma}_2\cdot\vec{n}) + A_6(\vec{\sigma}_1\cdot\vec{n})$$
$$+A_3(\vec{\sigma}_2\cdot\vec{n})(\vec{\sigma}_1\cdot\vec{n}) + A_4(\vec{\sigma}_2\cdot\vec{m})(\vec{\sigma}_1\cdot\vec{m}) + A_5(\vec{\sigma}_2\cdot\vec{l})(\vec{\sigma}_1\cdot\vec{l})\ , \tag{17}$$

where A_j denote the invariant amplitudes; $\vec{l},\ \vec{m}$ are the unite vectors along $\vec{p_2}$ and $\vec{\Delta}_T$, respectively, and $\vec{n} = \vec{l}\times\vec{m}$. The amplitude of elastic antiquark-nucleon scattering will be denoted by $\hat{B}$. It is related to the invariant amplitudes B_j by formula (17) in which we are to make the substitutions $A_j \to B_j,\ \vec{\sigma}_2 \to \vec{\sigma}_3,\ \vec{p_2} \to \vec{p_3}$. For pomeron exchanges $B_j = A_j$, relations between B_j and A_j for general case will be discussed later. In accordance with (1) we present A_j as a sum of amplitudes $A_j^{(n)}$ describing the n pomeron exchange contributions. Putting in (13) formulæ (12), (15), (16), (3), (4), (5) we get the expressions for amplitudes

$$A_j^{(n)} = C_{sh}^{(n)}[\eta_P(0)(s/s_0)^{\alpha_P(0)-1}]^n \exp\{-\lambda_P\vec{\Delta}_T^2/n\}a_j^{(n)}\ , \tag{18}$$

where the parameter λ_a for any reggeon a is given by

$$\lambda_a = \frac{r_a^2}{2} + \alpha'_a(0)[\ln(s/s_0) + \frac{\pi}{2}\sigma_a\eta_a^*(0)]\ . \tag{19}$$

The expressions for the nonzero amplitudes $a_j^{(n)}$ look like

$$a_1^{(1)} = p_sP_s\ ,$$
$$a_2^{(1)} = i\Delta_Tp_yp_s\ ,$$
$$a_6^{(1)} = i\Delta_TP_yP_s\ ,$$
$$a_3^{(1)} = -\Delta_T^2p_yP_y\ . \tag{20}$$

The formulæ for the two pomeron exchanges read

$$a_1^{(2)} = \frac{i}{4\lambda_P}\{(p_sP_s)^2 - \frac{1}{2\lambda_P}[(p_yP_s)^2+(p_sP_y)^2](\lambda_P\Delta_T^2/2-1)$$
$$+\frac{(p_yP_y)^2}{4\lambda_P^2}(\lambda_P^2\Delta_T^4/4-\lambda_P\Delta_T^2+2)\}\ ,$$
$$a_2^{(2)} = -\frac{\Delta_T}{8\lambda_P^2}p_yp_s[2\lambda_PP_s^2-P_y^2(\lambda_P\Delta_T^2/2-1)]\ ,$$
$$a_6^{(2)} = -\frac{\Delta_T}{8\lambda_P^2}P_yP_s[2\lambda_Pp_s^2-p_y^2(\lambda_P\Delta_T^2/2-1)]\ ,$$
$$a_3^{(2)} = -i\frac{\Delta_T^2}{8\lambda_P}p_yP_yp_sP_s\ . \tag{21}$$

For the three vacuum pole exchange we have

$$a_1^{(3)} = -\frac{1}{18\lambda_P^2}\{(p_sP_s)^3+\frac{1}{\lambda_P}(1-\lambda_P\Delta_T^2/3)[p_sp_y^2P_s^3+p_s^3P_sP_y^2]$$
$$+\frac{p_sp_y^2P_sP_y^2}{2\lambda_P^2}(3-\frac{4}{3}\lambda_P\Delta_T^2+\frac{2}{9}\lambda_P^2\Delta_T^4)\}\ ,$$
$$a_2^{(3)} = -i\frac{\Delta_T}{54\lambda_P^2}\{3p_s^2p_yP_s^3+\frac{1}{3\lambda_P}(2-\lambda_P\Delta_T^2/3)(p_y^3P_s^3+9p_s^2p_yP_sP_y^2)$$
$$-\frac{3}{\lambda_P}p_s^2p_yP_sP_y^2+\frac{p_y^3P_sP_y^2}{3\lambda_P^2}(\frac{9}{4}-\frac{2}{3}\lambda_P\Delta_T^2+\frac{2}{27}\lambda_P^2\Delta_T^4)\}\ ,$$
$$a_6^{(3)} = -i\frac{\Delta_T}{54\lambda_P^2}\{3p_s^3P_s^2P_y+\frac{1}{3\lambda_P}(2-\lambda_P\Delta_T^2/3)(p_s^3P_y^3+9p_sp_y^2P_s^2P_y)$$
$$-\frac{3}{\lambda_P}p_sp_y^2P_s^2P_y+\frac{p_sp_y^2P_y^3}{3\lambda_P^2}(\frac{9}{4}-\frac{2}{3}\lambda_P\Delta_T^2+\frac{2}{27}\lambda_P^2\Delta_T^4)\}\ ,$$
$$a_3^{(3)} = \frac{\Delta_T^2}{18\lambda_P^2}\{p_s^2p_yP_s^2P_y+\frac{1}{9\lambda_P}(2-\lambda_P\Delta_T^2/3)(p_y^3P_s^2P_y+p_s^2p_yP_y^3)\}$$
$$+\frac{p_y^3P_y^3}{972\lambda_P^5}\{4+\frac{13}{3}\lambda_P\Delta_T^2-\frac{8}{9}\lambda_P^2\Delta_T^4+\frac{2}{27}\lambda_P^3\Delta_T^6\}\ ,$$
$$a_4^{(3)} = \frac{p_y^3P_y^3}{486\lambda_P^5}(2+\frac{1}{6}\lambda_P\Delta_T^2)\ . \tag{22}$$

We can easily see from formulæ (17), (18), (20), (21) that at $\Delta_T=0$ all the spin-dependent amplitudes of the one and two pomeron exchange contributions vanish, hence there are no polarization phenomena for this case. Taking into account (7) we get from (17), (18) and (22) for $\Delta_T=0$

$$\hat{A}_P^{(3)} = -C_{sh}^{(3)}\eta_P^3(0)(s/s_0)^{3\alpha_P(0)-3}$$
$$\{\frac{P_s^6}{486\lambda_P^2}+\frac{5}{243\lambda_P^3}P_y^2P_s^4+\frac{1}{36\lambda_P^4}P_y^4P_s^2-\frac{P_y^6}{243\lambda_P^5}(\vec\sigma_{1T}\cdot\vec\sigma_{2T})\}\ , \tag{23}$$

where we make use of the short notation

$$(\vec\sigma_{1T}\cdot\vec\sigma_{2T}) \equiv (\vec\sigma_1\cdot\vec m)(\vec\sigma_2\cdot\vec m)+(\vec\sigma_1\cdot\vec n)(\vec\sigma_2\cdot\vec n)=(\sigma_{1x}\cdot\sigma_{2x})+(\sigma_{1y}\cdot\sigma_{2y})\ .$$

We shall show that the last term in (23) gives the longitudinal spin-spin asymmetry in the γp collisions. Indeed, the wave function of the $q\bar{q}$-pair produced by the virtual photon with helicity $m = \pm 1$ looks like [4]

$$\Psi_\gamma^{(m)}(\vec{k}_T, z) = \frac{(\vec{k}_T \cdot \vec{e}^{(m)})[(2\lambda_2)(1-2z) - m]}{Q^2 z(1-z) + \vec{k}_T^2 + \mu_q^2} \delta^{\lambda_2, -\lambda_3} , \tag{24}$$

where λ_2 and λ_3 denote the helicities of the quark and antiquark, respectively. Vectors $\vec{e}^{(m)}$ of the photon polarization are $\vec{e}^{(\pm 1)} = (\vec{e}_x \pm i\vec{e}_y)/\sqrt{2}$, $\vec{e}_x$, $\vec{e}_y$ being unite vectors orthogonal to the z-axis directed along the photon three-momentum, $q^2 = -Q^2$ is the square of the heavy photon four-momentum, z denotes the photon three momentum fraction carried by the quark (more precisely $z = (p_2^0 + p_2^3)/(q^0 + q^3)$ where p_2^0 (q^0) denotes the quark (photon) energy and p_2^3 (q^3) is the z-component of the quark (photon) three-momentum), $\vec{k}_T$ is the transverse part of the quark momentum $\vec{p}_2$ before scattering and μ_q denotes the mass of the constituent quark. We can neglect μ_q except in the case when k_T^2 and $Q^2 z(1-z)$ are small ($\ll 1$ (GeV/c)^2). The density matrix of the $q\bar{q}$-pair corresponding to the wave function (24) looks like

$$\begin{aligned} \rho_{23}^{(m)} &= \frac{V}{4}\{I + (\vec{\xi}_2 \cdot \vec{\sigma}_2) + (\vec{\xi}_3 \cdot \vec{\sigma}_3) + \eta_{lj}\sigma_{2l}\sigma_{3j}\} , \\ V &= 2\vec{k}_T^2[z^2 + (1-z)^2]/[Q^2 z(1-z) + \vec{k}_T^2 + \mu_q^2]^2 . \end{aligned} \tag{25}$$

The nonzero components of $\vec{\xi}_2$, $\vec{\xi}_3$, η_{lj} are

$$\begin{aligned} \xi_{2z} = -\xi_{3z} &= \frac{m(2z-1)}{z^2 + (1-z)^2} , \\ \eta_{xx} = \eta_{yy} = \frac{2z(1-z)}{z^2 + (1-z)^2} &, \quad \eta_{zz} = -1 . \end{aligned} \tag{26}$$

If we put in (25) $V = 1$ we get the spin density matrix of the $q\bar{q}$-pair normalized to unity. The general form at $\Delta_T = 0$ of the spectator amplitude of $q\bar{q}$-pair scattering on the nucleon shown in Figs. 1a, 1b is

$$\begin{aligned} M_{sp}^q(0) = ee_q\{&A_1 + B_1 + A_4(\vec{\sigma}_{1T} \cdot \vec{\sigma}_{2T}) + B_4(\vec{\sigma}_{1T} \cdot \vec{\sigma}_{3T}) \\ &+ A_5(\vec{l} \cdot \vec{\sigma}_1)(\vec{l} \cdot \vec{\sigma}_2) + B_5(\vec{l} \cdot \vec{\sigma}_1)(\vec{l} \cdot \vec{\sigma}_3)\} , \end{aligned} \tag{27}$$

where e denotes the electric charge of the positron, ee_q is the electric charge of a quark and A_j, B_j are the amplitudes of quark-nucleon and antiquark-nucleon scattering, respectively. For the pomeron exchange amplitudes given by relations (17), (18), (20), (21), (22) $A_5 = B_5 = 0$ but they are nonzero if we include in the consideration secondary Regge trajectories or take into account four, five, etc. pomeron exchanges.

Our amplitudes are normalized so that the differential cross section of elastic scattering of unpolarized quarks on unpolarized nucleons is given by

$$\frac{d\sigma}{dt} = 4\pi \sum_{j=1,6} |A_j|^2 ,$$

hence the cross section for scattering of $q\bar{q}$-pair on the nucleon is

$$\frac{d\sigma}{dtdM_X^2} = 4\pi n_c \sum_{q=u,d,...} \int tr\{(M_{sp}^q)^+(0) M_{sp}^q(0) \rho_{23}^{(m)} \rho_1\} \delta\Big(M_X^2 - \frac{\vec{k}_T^2 + \mu_q^2}{z(1-z)}\Big) dz d^2\vec{k}_T \ , \quad (28)$$

where $M_{sp}^+(0)$ denotes a hermitian conjugate quantity, $n_c = 3$ is the number of the quark colours, M_X denotes the mass of the $q\bar{q}$-pair in the final state and ρ_1 is the spin density matrix of the proton with the longitudinal polarization $\vec{\zeta}_P = (0, 0, \zeta_P)$

$$\rho_1 = \frac{1}{2}(I + \vec{\zeta}_P \cdot \vec{\sigma}_1) \ . \quad (29)$$

Putting (25), (26), (27), (29) into (28) and integrating over $d^2\vec{k}_T$ which gives due to the δ-function $\vec{k}_T^2 = M_0^2 z(1-z)$ we get

$$\begin{aligned}\frac{d\sigma}{dtdM_X^2} = 8\pi^2 \frac{e^2 e_q^2 n_c M_X^2}{(Q^2 + M_X^2)^2} \sum_{q=u,d,...} \int_0^1 \Big\{ & [|A_1 + B_1|^2 + 2|A_4|^2 + 2|B_4|^2 \\ & + |A_5|^2 + |B_5|^2 - 2\Re(A_5 B_5^*)][z^2 + (1-z)^2] + 8\Re(A_4 B_4^*) z(1-z) \\ & + 2m\zeta_P(2z-1)[|B_4|^2 - |A_4|^2 + \Re((A_1 + B_1)(A_5^* - B_5^*))] \Big\} M_0^2 \theta(M_0^2) dz \quad (30)\end{aligned}$$

where $M_0^2 = M_X^2 - \mu_q^2/[z(1-z)]$ and $\theta(x)$ denotes the Heavyside function ($\theta(x) = 1$ for $x \geq 0$ otherwise $\theta(x) = 0$). The formula for the longitudinal spin-spin asymmetry A_{LL} follows from its definition

$$A_{LL} = \{\sigma(+,+) - \sigma(+,-)\}/\{\sigma(+,+) + \sigma(+,-)\} \ , \quad (31)$$

where we have applied the short notation

$$\sigma(+,\pm) = \frac{d\sigma(m = 1, \zeta_P = \pm 1)}{dtdM_X^2} \ ,$$

and relation (30)

$$\begin{aligned}A_{LL} = 2 \sum_{q=u,d,...} \int_0^1 & (2z-1)[|B_4|^2 - |A_4|^2 + \Re((A_1 + B_1)(A_5^* - B_5^*))] M_0^2 \theta(M_0^2) dz \\ & / \sum_{q=u,d,...} \int_0^1 \Big\{ [|A_1 + B_1|^2 + 2|A_4|^2 + 2|B_4|^2 + |A_5|^2 + |B_5|^2 \\ & - 2\Re(A_5 B_5^*)][z^2 + (1-z)^2] + 8\Re(A_4 B_4^*) z(1-z) \Big\} M_0^2 \theta(M_0^2) dz \ . \quad (32)\end{aligned}$$

We see from (32) that the asymmetry does not depend on Q^2. If we consider a kinematics for which $k_T \gg m_q$ we can omit m_q in (24). For this case $M_0^2 = M_X^2$ in (30), (32) and A_{LL} does not depend on M_0^2 too. But these properties take place for the spectator graphs only (shown in Fig. 1a, 1b). We see also from (32) that the order of magnitude of A_{LL} for the case under consideration when $A_5 = B_5 = 0$ is

$$A_{LL} \sim |A_4|^2/|A_1|^2 \sim \frac{P_y^{12}}{P_s^{12}|\lambda_P|^6} \leq 10^{-12}. \quad (33)$$

For our estimate of A_1 we have used the first term in (23) and for A_4 we have used the last term. We can see from (19) that $|\lambda_P|$ increases with s. If we make use of the value for $|\lambda_P|$ at $s = s_0$ and take the values of P_y and P_s from Table 1 we get the numerical estimate given by relation (33). The authors of [11], [12] stated that the value $P_y = 0$ is compatible with experimental data. Considering the value of P_y presented in Table 1 as one standard deviation we have for $\tilde{P}_y = 3P_y$ (three standard deviations) instead of (33) an estimate $A_{LL} \leq 10^{-6}$. As $\lambda_P \sim \ln s$ in accordance with (19) at asymptotically high energy, then $A_{LL} \sim \ln^{-6} s_{\gamma p}$ where $s_{\gamma p}$ is the square of the photon-nucleon center-of-mass energy. We would like to make some remarks. It is easy to see from formula (32) that the numerator does not vanish. Indeed, the amplitude A_4 depends on z as $s_{12} = (p_1 + p_2)^2 \approx z s_{\gamma p}$ and B_4 depends on $(1-z)$ as $s_{13} = (p_1+p_3)^2 \approx (1-z)s_{\gamma p}$. Due to the positive signature of the pomeron the amplitude for quark-nucleon scattering is equal to the antiquark-nucleon amplitude at the same collision energy. Hence the quantity $M_0^2(2z-1)[|B_4|^2 - |A_4|^2]$ does not change sign after the replacement $z \rightarrow (1-z)$ and the numerator in (32) is not equal to zero if A_4 and B_4 depend on z. In all our further considerations of properties of some quantities under the transformation $z \rightarrow (1-z)$ we shall omit M_0^2 as it is invariant under this transformation. When A_4 and B_4 are independent of the collision energy, then $A_{LL} = 0$. But we can see from (18), (20), (21), (22) that if $\alpha_P(0) = 1$, then the dependence of the amplitudes A_j on z is due to the dependence of λ_P on s which is logarithmic in accordance with (19). In reality (see Table 1) $\alpha_P(0)$ is very close to unity and the s-dependence of the amplitudes of the vacuum pole exchange is rather feeble. Therefore we have an additional suppression compared to our rough estimate (33). Hence we conclude that in the high energy limit when contributions of all known Regge trajectories except the vacuum one are suppressed, the spin-spin asymmetry in the diffractive dissociation of the virtual photon with high Q^2 is much less than 10^{-6} for the spectator diagram contribution. In our simple model with parameters extracted from the experimental data on hadron-hadron scattering at energies 10 - 100 GeV we have an even lower limit at $\Delta_T = 0$ $A_{LL} \leq 10^{-12}$.

3 Spectator graphs. Secondary Regge trajectory contributions

Let us consider the contributions of the ρ, f, A_2, ω reggeons. It is easy to see from Table 1 that for any reggeon $c \neq P$ $\alpha_P(t) - \alpha_c(t) \approx 0.5$, hence the amplitude of exchange with a reggeon c is proportional to the small parameter $\varepsilon = \sqrt{s_0/s_{\gamma p}}$ compared with the pomeron exchange amplitude. We remind that $s_{\gamma p}$ is the square of the center-of-mass energy of the γp-system. We decompose all amplitudes, cross sections and asymmetries into power a series in ε up to ε^2 terms included. Now the amplitude of quark-nucleon scattering looks like [14]

$$\hat{A} = \sum_n \hat{A}_P^{(n)} + \frac{1}{1!} \sum_{n,c} \hat{A}_{c,P}^{(n-1)} + \frac{1}{2!} \sum_{n,c,d} \hat{A}_{c,d,P}^{(n-2)} + \cdots \tag{34}$$

with $c,\ d\ \neq P$. In (34) $\hat{A}^{(n-1)}_{c,P}$ denotes the amplitude of exchanges with a reggeon c and $n-1$ pomerons, and the amplitude $\hat{A}^{(n-2)}_{c,d,P}$ describes contributions of reggeons c, d and $n-2$ pomerons. The formulæ for the amplitudes $\hat{A}^{(n-1)}_{c,P}$ and $\hat{A}^{(n-2)}_{c,d,P}$ read [14]

$$\begin{aligned}
\hat{A}^{(n-1)}_{c,P} &= \frac{i^{n-1}}{\pi^{n-1}(n-1)!}\int N_1^n(c)N_2^n(c)G_c(\vec{\Delta}_1,s)G_P(\vec{\Delta}_2,s)...G_P(\vec{\Delta}_n,s) \\
&\qquad \delta(\sum_{i=1}^{n}\vec{\Delta}_i-\vec{\Delta})d^2\vec{\Delta}_1 d^2\vec{\Delta}_2...d^2\vec{\Delta}_n\ , \\
\hat{A}^{(n-2)}_{c,d,P} &= \frac{i^{n-1}}{\pi^{n-1}(n-2)!}\int N_1^n(c,d)N_2^n(c,d)G_c(\vec{\Delta}_1,s)G_d(\vec{\Delta}_2,s)G_P(\vec{\Delta}_3,s)...G_P(\vec{\Delta}_n,s) \\
&\qquad \delta(\sum_{i=1}^{n}\vec{\Delta}_i-\vec{\Delta})d^2\vec{\Delta}_1 d^2\vec{\Delta}_2...d^2\vec{\Delta}_n\ ,
\end{aligned} \tag{35}$$

with the n reggeon vertices given by

$$\begin{aligned}
N_1^n(c) &= C_{sh}^{(n)}\{C(\Delta_1)P(\Delta_2)...P(\Delta_n)\}\ , \\
N_2^n(c) &= \{c(\Delta_1)p(\Delta_2)...p(\Delta_n)\}\ , \\
N_1^n(c,d) &= C_{sh}^{(n)}\{C(\Delta_1)D(\Delta_2)P(\Delta_3)...P(\Delta_n)\}\ , \\
N_2^n(c,d) &= \{c(\Delta_1)d(\Delta_2)p(\Delta_3)...p(\Delta_n)\}\ ,
\end{aligned} \tag{36}$$

where $b(\Delta)$, $B(\Delta)$ ($b=c,\ d;\ B=C,\ D$) in (36) are the vertex functions of a reggeon b analogous to $p(\Delta)$, $P(\Delta)$ defined in (3), (5), respectively. Relation (7) is valid for any reggeon with isospin $T=0$. For the ρ and A_2 mesons having isospin $T=1$ the vertices contain the isospin Pauli matrices τ_j. For example, the vertex for the emission of the ρ_j meson ($\rho^{\pm}=(\rho_1\pm i\rho_2)/\sqrt{2},\ \rho^0=\rho_3$) is proportional to τ_j. One can easily get the relation

$$b_s = B_s\ ,\quad b_y=\frac{3}{5}B_y \tag{37}$$

for the case $T=1$ with the aid of the wave functions (6). Formulæ for the antiquark-nucleon scattering can be obtained from (35), (36) with the substitutions $\hat{A}^{(n-1)}_{c,P}\rightarrow\hat{B}^{(n-1)}_{c,P}$, $\hat{A}^{(n-2)}_{c,d,P}\rightarrow\hat{B}^{(n-2)}_{c,d,P}$ the vertex functions $b(\Delta_1)$ (with $b=c,\ d$) being multiplied by the factor $\sigma_b(-1)^{T_b}$ where σ_b, T_b denote signature and isospin of reggeon b.

Let us consider the first order contributions ($\sim\varepsilon$) at $\vec{\Delta}_T=0$. There is only one nonvanishing amplitude A_1 of the one reggeon exchange. Hence the inclusion of the pole contributions of secondary Regge trajectories does not lead to any polarization phenomena. It follows from the general formulæ presented in Appendix that the two reggeon exchange amplitudes $A_3^{(2)}$, $A_4^{(2)}$ at $\vec{\Delta}_T=0$ looks like

$$A_3^{(2)}=A_4^{(2)}=-\frac{i\eta_c(0)\eta_P(0)}{2(\lambda_c+\lambda_P)^2}\Big(\frac{s}{s_0}\Big)^{\alpha_c(0)+\alpha_P(0)-2}(c_y p_s-p_y c_s)(C_y P_s-P_y C_s)C_{sh}^{(2)}\ , \tag{38}$$

where $c_s,\ c_y,\ C_s\equiv C_s(0),\ C_y\equiv C_y(0)$ are the vertex constants for a reggeon c which are analogous to $p_s,\ p_y,\ P_s(0),\ P_y(0)$ for the pomeron defined by (3), (5) (We denote vertices with the same letter as the letter denoting a reggeon. For example, the vertex constants for the ω reggeon will be denoted $\omega_s,\ \omega_y,\ \Omega_s,\ \Omega_y$ etc.).

As for the secondary trajectories $\alpha_c(0) \approx 0.5$ it follows from (38) that even for $\alpha_P(0) = 1$ $A_4^{(cP)} \sim \sqrt{s_0/s} \sim z^{-0.5}$ and for the antiquark-nucleon scattering amplitude $B_4^{(cP)} \sim (1-z)^{-0.5}$. We denote by $A_4^{(cP)}$ ($B_4^{(cP)}$) the part of the amplitude A_4 describing exchanges with the reggeons c and P. Hence the numerator in (32) does not vanish as the expression $(2z-1)(|B_4|^2-|A_4|^2)$ conserves its sign under the transformation $z \to (1-z)$. For reggeons with negative signature σ (ρ and ω reggeons), their contribution of the first order in ε to the numerator in (32) is equal to zero. For example, considering pomeron and ω exchanges we have $A_4^{(2)}(z) = A_4^{(PP)}(z) + A_4^{(\omega P)}(z)$ and $B_4^{(2)}(1-z) = A_4^{(PP)}(1-z) - A_4^{(\omega P)}(1-z)$. As the first order contribution to $(2z-1)(|B_4|^2 - |A_4|^2)$ equal to

$$2(2z-1)\{\Re[A_4^{*(PP)}(z)A_4^{(\omega P)}(z)] + \Re[A_4^{*(PP)}(1-z)A_4^{(\omega P)}(1-z)]\}$$

is an odd function under transformation $z \to (1-z)$, hence after integration over z we get a zero contribution to the numerator in (32). The general formulæ for the two and three reggeon exchange amplitudes presented in Appendix show that the contribution to A_5 of the first order in ε is equal to zero.

We start discussion of terms of the second order in ε with consideration of the contribution of the amplitudes $A_5^{(3)}$ and $B_5^{(3)}$ in (32) as the two reggeon exchange amplitudes $A_5^{(2)}$ and $B_5^{(2)}$ vanish at $\Delta_T = 0$. We can see from the general formulæ presented in Appendix that the amplitude $A_5^{(3)}$ can be nonzero if and only if two exchanged reggeons (say c and h) have isospins equal to 1 (the third reggeon is the pomeron as we consider contributions of the second order in ε). Indeed, the formula for $A_5^{(3)}$ at $\vec{\Delta}_T = 0$ reads

$$A_5^{(3)} = \tilde{A}_5^{(3)}(\vec{\tau}_1 \cdot \vec{\tau}_j) \ , \tag{39}$$

where $\vec{\tau}_1$ and $\vec{\tau}_j$ are the Pauli matrices acting on the isospin variables of the nucleon and quark ($j = 2$) or antiquark ($j = 3$) and $\tilde{A}_5^{(3)}$ is

$$\tilde{A}_5^{(3)} = \frac{\eta_P(0)\eta_C(0)\eta_H(0)}{9(\lambda_P\lambda_C + \lambda_P\lambda_H + \lambda_C\lambda_H)^2}\Big(\frac{s}{s_0}\Big)^{\alpha_P(0)+\alpha_C(0)+\alpha_H(0)-3} p_y P_y c_y C_y h_y H_y$$
$$\Big\{\frac{C_s}{C_y}\Big(\frac{c_s}{c_y} - \frac{p_s}{p_y}\Big) + \frac{H_s}{H_y}\Big(\frac{h_s}{h_y} - \frac{p_s}{p_y}\Big) + \frac{c_s}{c_y}\Big(\frac{H_s}{H_y} - \frac{P_s}{P_y}\Big) + \frac{h_s}{h_y}\Big(\frac{C_s}{C_y} - \frac{P_s}{P_y}\Big) + \frac{p_s P_s}{p_y P_y}\Big\} C_{sh}^{(3)} \ . \tag{40}$$

If a reggeon c has the same signature as h, then the amplitude $\tilde{A}_5^{(3)}$ is invariant under the charge conjugation transformation $c_s \to \pm c_s,\ c_y \to \pm c_y,\ h_s \to \pm h_s,\ h_y \to \pm h_y$. We do not consider in the present paper charge exchange of the proton, hence we can write $(\tau_{1z}\tau_{jz})$ instead of $(\vec{\tau}_1 \cdot \vec{\tau}_j)$ in (39). Acting on the quark ($q = u,\ d$) and antiquark τ_{jz} gives values with opposite signs therefore $B_5^{(3)}(s) = -A_5^{(3)}(s)$. Returning to (32) we see that

$$\begin{aligned} &(2z-1)\Re\{(A_1^* + B_1^*)(A_5 - B_5)\} \\ &= (2z-1)\Re\{[A_1^*(z) + A_1^*(1-z)][\tilde{A}_5^{(3)}(z) + \tilde{A}_5^{(3)}(1-z)]\} \end{aligned} \tag{41}$$

is an odd function with respect to the transformation $z \to 1-z$, hence the integral over z for term (41) in the numerator in (32) vanishes. We have taken into account in (41)

that A_1 contains pomeron exchanges only as $\tilde{A}_5^{(3)},\ \tilde{B}_5^{(3)} \sim \varepsilon^2$. It is easy to conclude that the amplitudes contribute to the spin-spin asymmetry if they contain secondary reggeon exchanges with opposite sign signatures. For the Regge trajectories considered in the present paper such an amplitude is the amplitude of $P\rho A_2$ exchange.

The second order contributions of the difference $|B_4|^2 - |A_4|^2$ to the numerator in (32) which do not vanish after integration over z can be divided into two groups. The first one

$$\begin{aligned} &|A_4^f(1-z) + A_4^{A_2}(1-z)|^2 - |A_4^f(z) + A_4^{A_2}(z)|^2 \\ &+|A_4^\rho(1-z) + A_4^\omega(1-z)|^2 - |A_4^\rho(z) + A_4^\omega(z)|^2 \end{aligned} \tag{42}$$

contains squares of first order amplitudes where A_4^f denotes the part of the amplitude A_4 describing exchange with the reggeon f and some number of pomeron exchanges. The amplitudes $A_4^{A_2},\ A_4^\rho,\ A_4^\omega$ have an analogous meaning. Relation (42) shows that the $f,\ \rho,\ \omega,\ A_2$ reggeons contribute to the spin-spin asymmetry, the interference terms for reggeons with positive ($f,\ A_2$) and negative ($\rho,\ \omega$) signatures are absent. The second group of contributions to the numerator in (32) not vanishing after integration over z looks like

$$\begin{aligned} &\Re\Big\{A_4^{*P}(1-z)[A_4^{\rho\rho}(1-z) + A_4^{ff}(1-z) + A_4^{\omega\omega}(1-z) \\ &\qquad + A_4^{A_2A_2}(1-z) + A_4^{\rho\omega}(1-z) + A_4^{fA_2}(1-z)\Big\} \\ &-\Re\Big\{A_4^{*P}(z)[A_4^{\rho\rho}(z) + A_4^{ff}(z) + A_4^{\omega\omega}(z) + A_4^{A_2A_2}(z) + A_4^{\rho\omega}(z) + A_4^{fA_2}(z)\Big\}\ . \end{aligned} \tag{43}$$

Expression (43) represents the interference term of the amplitude A_4 of the second order in ε with A_4^P which contains pomeron exchanges only and starts with the three pomeron exchange amplitude as has been explained above. As for expression (42), the contributions of exchanges of two secondary Regge trajectories (and some number of pomeron exchanges) with positive and negative signatures vanish after integration over z.

4 Contributions of nonspectator graphs to A_{LL}

The amplitude of the non-spectator graphs shown in Fig. 2a and 2b can be written as a sum of two terms $F_{es}^q(\vec{\Delta}) + F_{che}^q(\vec{\Delta})$ where

$$\begin{aligned} F_{es}^q(\vec{\Delta}) = ee_q\frac{i}{2\pi}\int &[\hat{A}_{m_q}(\vec{\Delta}_1)\hat{B}_{-m_q}(\vec{\Delta}_2) + \hat{B}_{-m_q}(\vec{\Delta}_2)\hat{A}_{m_q}(\vec{\Delta}_1) + \hat{D}(\vec{\Delta}_1,\vec{\Delta}_2)] \\ &\psi_\gamma^{(m)}(\vec{k}_T + \vec{\Delta}_2, z)\delta(\vec{\Delta} - \vec{\Delta}_1 - \vec{\Delta}_2)d^2\vec{\Delta}_1 d^2\vec{\Delta}_2\ , \end{aligned} \tag{44}$$

$$\begin{aligned} F_{che}^q(\vec{\Delta}) = e\tilde{e}_q\frac{i}{2\pi}\int &[\hat{A}_{che}(\vec{\Delta}_1)\hat{B}_{che}(\vec{\Delta}_2)(2-4m_q) + \hat{B}_{che}(\vec{\Delta}_2)\hat{A}_{che}(\vec{\Delta}_1)(2+4m_q) \\ &+\hat{E}(\vec{\Delta}_1,\vec{\Delta}_2)]\psi_\gamma^{(m)}(\vec{k}_T + \vec{\Delta}_2, z)\delta(\vec{\Delta} - \vec{\Delta}_1 - \vec{\Delta}_2)d^2\vec{\Delta}_1 d^2\vec{\Delta}_2\ , \end{aligned} \tag{45}$$

In (44) $F_{es}^q(\vec{\Delta})$ is the amplitude of elastic scattering both of the quark with flavour q and its antiquark on the proton and $F_{che}^q(\vec{\Delta})$ in (45) describes quark charge exchange ($u\bar{u} \to d\bar{d}$ or $d\bar{d} \to u\bar{u}$) with the same flavour q in the final state as for elastic scattering.

We remind that charge exchange of the proton is not considered here. The electric charge of the final quark is related to the third component of its isospin m_q by the formula

$$e_q = \frac{1}{6} + m_q \ . \tag{46}$$

The electric charge of the initial quark before charge exchange process is equal to

$$\tilde{e}_q = \frac{1}{6} - m_q \ . \tag{47}$$

Let us compare the graphs shown in Fig. 1a and Fig. 2a. The transverse component of the initial quark momentum in Fig. 1a is $\vec{k}_T$ and the final transverse momentum is equal to $\vec{k}_T + \vec{\Delta}$. The final momenta for both graphs in Fig. 1a and Fig. 2a are equal to each other. Hence the transverse component of the initial quark momentum in Fig. 2a is $\vec{k}_T + \vec{\Delta} - \vec{\Delta}_1 = \vec{k}_T + \vec{\Delta}_2$ which is the first argument of the photon wave function $\psi_\gamma^{(m)}$ for the nonspectator diagrams shown in Fig. 2a and Fig. 2b. When we consider exchanges of reggeons with isospin $T = 1$, then amplitude of elastic qN scattering becomes an operator in isospin space

$$\hat{A} = \hat{A}_s + \hat{A}_{che}(\vec{\tau}_1 \cdot \vec{\tau}_2) \ . \tag{48}$$

For elastic quark-proton scattering the amplitude looks like

$$\hat{A}_{m_q} = \hat{A}_s + \hat{A}_{che}(2m_q) \tag{49}$$

and for antiquark elastic scattering on the proton it is described by the formula

$$\hat{B}_{-m_q} = \hat{B}_s + \hat{B}_{che}(-2m_q) \tag{50}$$

as the third component of the antiquark isospin is equal to $-m_q$. We remind that $\hat{B}_s$ and $\hat{B}_{che}$ can be obtained from $\hat{A}_s$ and $\hat{A}_{che}$ by multiplying every quark-quark-reggeon vertex by the factor $\sigma_b(-1)^{T_b}$ for a reggeon b. Relations (46), (47), (48), (49), (50) explain the meaning of all quantities in the main formulæ (44), (45) except $\hat{D}(\vec{\Delta}_1, \vec{\Delta}_2)$ and $\hat{E}(\vec{\Delta}_1, \vec{\Delta}_2)$ which are presented in Appendix. They are equal to zero for two reggeon exchanges. Their existence is related with the fact that the amplitudes of three and more reggeon exchanges cannot be presented even at fixed Δ_1 and Δ_2 as a sum of products of the qN and $\bar{q}N$ amplitudes (the first two terms in the brackets in (44) and (45)) multiplied by the wave function of the $q\bar{q}$-pair. To understand why this is so let us consider, for example, the graph (in the eikonal approximation) shown in Fig. 4a. It is obvious, that the amplitude of such a subprocess with fixed $\vec{\Delta}_1$ and $\vec{\Delta}_2$ can be presented as a product of the qN and $\bar{q}N$ amplitudes (multiplied by the photon wave function). After some permutation of vertices along the proton and quark lines we can get the graph shown in Fig. 4b. The symmetry property of many reggeon emission vertices implies an existence of the graph shown in Fig. 4b. But the latter diagram is irreducible and cannot be presented for fixed momenta $\vec{\Delta}_1$ and $\vec{\Delta}_2$ as a product of the qN and $\bar{q}N$ amplitudes and the photon wave function.

We start our discussion of properties of the nonspectator diagram contributions to the spin-spin asymmetry with the case of two reggeon exchanges. Formulæ (44) and

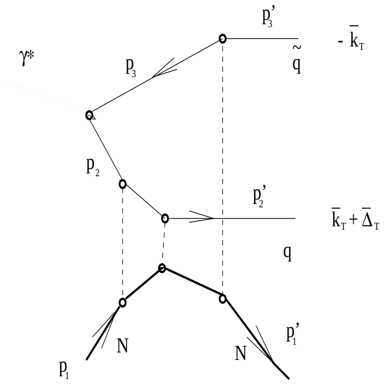


Fig. 4a: Non-spectator graph in eikonal approximation with three reggeon exchanges. Lines have the same meaning as in Fig. 1a.

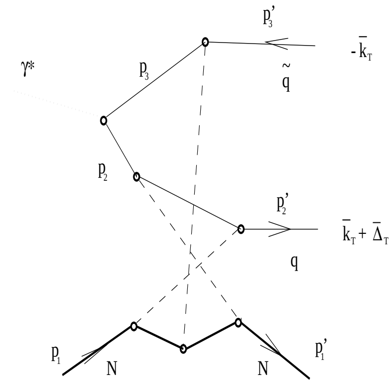


Fig. 4b: Non-spectator graph obtained from Fig. 4a after permutation of vertices. Lines have the same meaning as in Fig. 1a.

(45) show that due to the integration over $\vec{\Delta}_1$ and $\vec{\Delta}_2$ the factorization into the qN, $\bar{q}N$ amplitudes and the wave function $\psi_\gamma^{(m)}$ which has been used for the case of the spectator graphs is lost. For $\vec{\Delta}_T = 0$ we have $\vec{\Delta}_1 = -\vec{\Delta}_2$ in (44), (45) and integration runs over typical momentum transfers of soft scattering, $\vec{\Delta}_1^2 \sim 1/|\lambda_P| \leq 1$ (GeV/c)^2. If we consider large Q^2 and k_T^2 ($Q^2 \gg 1$ (GeV/c)^2, $k_T^2 \geq k_{min}^2 \gg 1$ (GeV/c)$^2 \gg m_q^2$), then for not too low $z(1-z)$ we can omit $\vec{\Delta}_2$ and m_q ($k_{min} \gg m_q$) in $\psi_\gamma^{(m)}(\vec{k}_T + \vec{\Delta}_2, z)$ as is easy to see from (24). For this case we present F_{es}^q by the approximate formula

$$F_{es}^q(0) = iee_q \hat{f} \psi_\gamma(\vec{k}_T, z) \ , \tag{51}$$

and for $\hat{f}$ it is easy to get the following relation from (44) and (17)

$$\begin{aligned} \hat{f} &= f_1 + f_2(\vec{\sigma}_{2T} \cdot \vec{\sigma}_{3T}) + f_3(\vec{\sigma}_{2T} \cdot \vec{\sigma}_{1T}) + f_4(\vec{\sigma}_{3T} \cdot \vec{\sigma}_{1T}) \\ &+ f_5(\vec{\sigma}_2 \cdot \vec{l})(\vec{\sigma}_3 \cdot \vec{l}) + f_6(\vec{\sigma}_2 \cdot \vec{l})(\vec{\sigma}_1 \cdot \vec{l}) + f_7(\vec{\sigma}_3 \cdot \vec{l})(\vec{\sigma}_1 \cdot \vec{l}) \ , \end{aligned} \tag{52}$$

where f_j are related to A_i and B_i as

$$\begin{aligned} f_1 &= \int_0^\infty [A_1(\Delta_1)B_1(\Delta_1) - A_6(\Delta_1)B_6(\Delta_1)] d\Delta_1^2 \ , \\ f_2 &= \frac{1}{2}\int_0^\infty [A_3(\Delta_1)B_3(\Delta_1) + A_4(\Delta_1)B_4(\Delta_1) - A_2(\Delta_1)B_2(\Delta_1)] d\Delta_1^2 \ , \\ f_3 &= \frac{1}{2}\int_0^\infty [A_3(\Delta_1)B_1(\Delta_1) + A_4(\Delta_1)B_1(\Delta_1) - A_2(\Delta_1)B_6(\Delta_1)] d\Delta_1^2 \ , \\ f_4 &= \frac{1}{2}\int_0^\infty [A_1(\Delta_1)B_3(\Delta_1) + A_1(\Delta_1)B_4(\Delta_1) - A_6(\Delta_1)B_2(\Delta_1)] d\Delta_1^2 \ , \\ f_5 &= \int_0^\infty A_5(\Delta_1)B_5(\Delta_1) d\Delta_1^2 \ , \\ f_6 &= \int_0^\infty A_5(\Delta_1)B_1(\Delta_1) d\Delta_1^2 \ , \\ f_7 &= \int_0^\infty A_1(\Delta_1)B_5(\Delta_1) d\Delta_1^2 \ . \end{aligned} \tag{53}$$

We have omitted the flavour index q in f, f_j, A_j, and B_j to have simpler notation. Formula (51) shows that the factorization under discussion is restored and we can use formulæ (28), (25), (26), (31) to get the spin-spin asymmetry given by the nonspectator graphs only

$$A_{LL} = 2\sum_{q=u,d,\ldots}\int_0^1 (2z-1)\Big\{|f_4|^2 - |f_3|^2 + \Re[(f_1-f_5)(f_6-f_7)^*]\Big\}\theta(M_1^2)dz$$
$$/\sum_{q=u,d,\ldots}\int_0^1 \Big\{|f_1|^2+4|f_2|^2+2|f_3|^2+2|f_4|^2+|f_5|^2$$
$$+|f_6|^2+|f_7|^2-\Re(f_1f_5^*+f_6f_7^*)][z^2+(1-z)^2]$$
$$+4\Re(f_1f_2^*-f_5f_2^*+f_3f_4^*)z(1-z)\}\theta(M_1^2)dz \qquad (54)$$

with $M_1^2 = M_X^2 - k_{min}^2/[z(1-z)]$. We shall discuss contributions to the numerator in (54). Putting (18) and (20) into (53) we obtain the relations

$$f_5 = f_6 = f_7 = 0 \qquad (55)$$

valid for the two reggeon exchange contributions. It follows from (55) and (54) that only f_3 and f_4 can contribute to the numerator of the expression for A_{LL} in the two reggeon exchange approximation for the nonspectator graph contributions. It is the contribution of f_3 and f_4 which will be discussed below. We have for the pure pomeron contributions in addition to (55) the relations

$$f_3 = f_4 = 0\,. \qquad (56)$$

Combining (55), (56) with (54) we conclude that the two pomeron exchanges do not contribute to A_{LL} as for the spectator graphs.

Let us apply more detailed notations for f_j. We denote by $f_j^{hc}(z)$ that part of the amplitude f_j which is determined by exchanges of some reggeons c and h. For example, for $f_6(z)$ we have from (53)

$$f_6^{hc}(z) = \int_0^\infty [A_5^c(z)B_1^h(1-z) + A_5^h(z)B_1^c(1-z)]d\Delta_1^2\,, \qquad (57)$$

where we have taken into account that the square of the center-of-mass energy for antiquark-proton scattering is $(1-z)s_{\gamma p}$ and hence B_j depends on $1-z$. We have omitted the Δ_1-dependence of A_j, B_j in (57) as this is not essential for our consideration now. Using the relation $B_j^a(z) = \sigma_a A_j^a(z)$ for any reggeon a we can easily get the relation of interest

$$f_3^{ch}(z) = \sigma_c\sigma_h f_4^{ch}(1-z) \qquad (58)$$

from (53). Let us consider the contributions of the first order amplitudes f_3^{cP}, f_3^{hP} and f_4^{cP}, f_4^{hP} to the numerator in (54) which looks like

$$\Re[f_4^{*cP}(z)f_4^{hP}(z) - f_3^{*cP}(z)f_3^{hP}(z)]\,. \qquad (59)$$

Applying (58) to the second term in (59) we transform it to the relation

$$\Re[f_4^{*cP}(z)f_4^{hP}(z) - \sigma_c\sigma_h f_4^{*cP}(1-z)f_4^{hP}(1-z)]\,. \qquad (60)$$

It follows from (60) and (54) that the integral over z vanishes when $\sigma_c\sigma_h = -1$ and it is generally speaking nonzero if $\sigma_c\sigma_h = 1$. As has been pointed out the amplitudes of pure pomeron exchanges f_3^{PP} and f_4^{PP} are equal to zero hence there are no contributions of the first order in ε to A_{LL} in the two reggeon exchange approximation. It follows from (60) and (54) that any secondary reggeon trajectory c contributes to the numerator of A_{LL} (the term with $h = c$ in (60)) but there are no interference terms for any two reggeons c and h with opposite signatures σ_c and σ_h. As can be seen from the general formulæ presented in the Appendix the contributions $\sim \varepsilon^0$ to f_3 and f_4 start from the three pomeron exchange contribution. Denoting such amplitudes as f_3^{PPP} and f_4^{PPP} we can easily check that their contribution to A_{LL} does not vanish. The first order contribution to A_{LL} due to the interference terms between the three pomeron exchange amplitudes f_3^{PPP}, f_4^{PPP} and f_3^{hP}, f_4^{hP} looks like

$$2\Re[f_4^{*PPP}(z)f_4^{hP}(z) - f_3^{*PPP}(z)f_3^{hP}(z)]\ . \tag{61}$$

Using (53) we can transform (61) to the relation

$$2\Re[f_4^{*PPP}(z)f_4^{hP}(z) - \sigma_h f_4^{*PPP}(1-z)f_4^{hP}(1-z)] \tag{62}$$

as (59) was transformed into (60). Formula (62) shows that contributions to A_{LL} of the first order in ε vanish for reggeons with negative signatures. It is easy to conclude that contributions of the second order due to interference terms $f_3^{*PPP}f_3^{hc}$ and $f_4^{*PPP}f_4^{hc}$ $(c,\ h \neq P)$ vanish if reggeons c and h have opposite signatures.

Comparison of (27) with (52) shows that the total amplitude F_{tot} of $q\bar{q}$-pair scattering on the proton for large Q^2 and k_T^2 at $\Delta_T = 0$ has the same form (52) as for $\hat{f}$ and we can formally take into account the spectator graph amplitude if we make the following replacements: $f_1 \to f_1 - iA_1 - iB_1,\ f_3 \to f_3 - iA_4,\ f_4 \to f_4 - iB_4,\ f_6 \to f_6 - iA_5,\ f_7 \to f_7 - iB_5$ and $f_2,\ f_5$ being unchanged. The main conclusion that there are no contributions of the first order to the numerator of expression (54) for the spin-spin asymmetry due to exchanges with a negative signature reggeon plus some number of pomerons remains valid. Contributions of the second order in ε of any signature reggeons are not suppressed but interference terms for opposite signature reggeon contributions to the numerator of A_{LL} are absent. In reality at $\Delta_T = 0$ the spin-dependent amplitude of $q\bar{q}$-pair scattering off the proton due to pomeron exchange is numerically very small. Indeed, the ratio of it to the spin-independent part of the pomeron exchange amplitude is proportional to a high power of the small quantity $P_y/(P_s\sqrt{|\lambda_P|})$ (compare for example the fourth term in (23) with the first one). Hence all interference terms in the numerator in (54) of the first order ($\sim \varepsilon$) with the spin-dependent amplitudes of pomeron exchanges can be at energies achieved experimentally up to now at HERA numerically much less than contributions of the second order which are not suppressed by some selection rules. As this is not excluded experimentally that $P_y = p_y = 0$, then in this case the lowest order contributions to A_{LL} are the second order ones.

Last but important remarks. If we restrict our consideration by demanding $k_T \geq k_{min}$, then due to the δ-function in (28) $z(1-z) \geq (k_{min}^2 + \mu_q^2)/M_X^2$ and we integrate in (28) and (54) over z from z_{min} up to $1 - z_{min}$ with $z_{min} \approx (k_{min}^2 + \mu_q^2)/M_X^2$. The square of the center-of-mass energy of qp or $\bar{q}p$ scattering is greater than $s_{min} \approx s_{\gamma p}(k_{min}^2 + \mu_q^2)/M_X^2$.

If $s_{min} \gg s_0$ we suppress the low energy parts of contributions to A_{LL}. But this is the region of integration over z where secondary reggeon trajectory contributions are most important. Hence increasing the value of k_{min} we can decrease the value of A_{LL} appreciably. If $s_{min} \leq s_0$ we are out of the applicability of the Regge phenomenology hence we should avoid kinematics with very small z. Integrals (28) and (54) are formally divergent at $z \to 0$ and $z \to 1$ as for a contribution of some secondary reggeon h with $\alpha_h(0) = 0.5$ to quark-proton scattering the energy dependence looks like $(zs_{\gamma p}/s_0)^{\alpha_h(0)-1} \sim z^{-1/2}$ and for antiquark-proton scattering the amplitude depends on z as $(1-z)^{\alpha_h(0)-1} \sim (1-z)^{-1/2}$. But the formula $z_{min} \approx (k_{min}^2 + \mu_q^2)/M_X^2$ shows that even for $k_{min} = 0$ $z_{min} > 0$ due to the mass of the constituent quark. Hence we cannot neglect μ_q in the kinematics with low or zero k_T to avoid the infrared divergence of the integrals for the cross section and for the longitudinal spin-spin asymmetry.

5 Numerical results and discussion

For the numerical calculation performed both for $\Delta_T^2 = 0$ and at nonzero momentum transfers we have not used the approximations applied in previous sections to discuss some qualitative properties of the longitudinal spin-spin asymmetry. For the amplitude corresponding to the spectator graphs $F_{sp}^q(\vec{\Delta}_T)$ we have used the formula

$$F_{sp}^q(\vec{\Delta}_T) = ee_q[\hat{A}_{m_q}(\vec{\Delta}_T)\Psi_\gamma^{(m)}(\vec{k}_T, z) + \hat{B}_{-m_q}(\vec{\Delta}_T)\Psi_\gamma^{(m)}(\vec{k}_T + \vec{\Delta}_T, z)] \qquad (63)$$

where $\hat{A}_{m_q}(\vec{\Delta})$ is the full amplitude of elastic quark-proton scattering containing six invariant amplitudes A_j defined in (17). The isospin structure of the amplitude is given by (48) and the meaning of the notation $\hat{A}_{m_q}(\vec{\Delta})$ is explained by relation (49). The amplitude of antiquark-proton scattering $\hat{B}_{-m_q}(\vec{\Delta})$ has been defined in (50). At $\Delta_T = 0$ formula (63) reduces to

$$F_{sp}^q(0) = M_{sp}^q(0)\Psi_\gamma^{(m)}(\vec{k}_T, z)$$

with $M_{sp}^q(0)$ defined in (27). For the contributions of the nonspectator graphs we have applied formulæ (44) and (45) without using the approximate relation (51). We put the total amplitude $F^q(\vec{\Delta}_T)$ for the diffractive production of the quark-antiquark pair with a flavour q in γp-scattering

$$F_q(\vec{\Delta}_T) = F_{sp}^q(\vec{\Delta}_T) + F_{es}^q(\vec{\Delta}_T) + F_{che}^q(\vec{\Delta}_T) \qquad (64)$$

into the formula for the cross section of production of a hadronic system with the total mass M_X

$$\frac{d\sigma_m}{dtdM_x^2} = 4\pi n_c \sum_{q=u,d,\ldots} \int F_q^+(\vec{\Delta}_T)F_q(\vec{\Delta}_T)\rho_1\delta\Big(M_X^2 - \frac{\mu_q^2 + [\vec{k}_T + \vec{\Delta}_T(1-z)]^2}{z(1-z)}\Big)dzd^2k_T \qquad (65)$$

where the proton spin density matrix ρ_1 has been defined in (29) and $F_{es}^q(\vec{\Delta}_T)$, $F_{che}^q(\vec{\Delta}_T)$ have been given by (44), (45). We have omitted in (64) an index m for all amplitudes but

kept it for the cross section where m denotes the virtual photon helicity. The longitudinal spin-spin asymmetry has been calculated with the aid of the general formula (31).

Fig. 5 shows the results of the calculation of A_{LL} when both the spectator and non-spectator graphs have been taken into account. We see that the pure pomeron part of the spin-spin asymmetry shown by the dash-dotted line is very small ($< 10^{-15}$ at $s_{\gamma p} \geq 100$ GeV^2). We put in the calculation the mass of the constituent quark $\mu_q = 0.35$ GeV/c^2, the minimal value of k_T equal to 0.2 GeV/c, the mass of the produced hadrons $M_X = 10$ GeV/c^2 and the heavy photon virtuality $Q^2 = 10$ (GeV/c)^2. The first order ($\sim \epsilon = \sqrt{s_0/s_{\gamma p}}$) contribution to A_{LL} is negative and it represents an interference term of the pomeron exchange amplitude with the amplitude of a sum of $\rho(770)$, $f_2(1270)$, $A_2(1320)$ and $\omega(782)$ exchanges (and some number of pomeron exchanges). Due to the smallness of the spin-dependent part of the pomeron exchange amplitude the first order contribution to A_{LL} (the dashed curve) is much smaller than the second order one shown with the dotted line in Fig. 5. We would like to remind that the phenomenological analysis of hadron-hadron scattering [11] and [12] is compatible with the zero value of the spin-dependent vertex of pomeron exchange. For this case the zeroth and first order contributions to A_{LL} vanish. But even if they do not vanish they are not important at $s_{\gamma p} \leq 9 \cdot 10^4$ GeV^2 (the highest energy of ep scattering at the collider HERA) since the second order contribution dominates very much as one can see from Fig. 5. The solid curve in Fig. 5 shows the behaviour of the longitudinal spin-spin asymmetry calculated with the aid of formulæ (63), (64), (44), (45), (65) and (31) without decomposition of A_{LL} into a power series in ϵ. We shall call such A_{LL} "the total asymmetry". We remind that the calculated amplitudes contain terms ϵ^0, ϵ, ϵ^2 only and we have taken into account one, two and three reggeon exchanges only.

At first sight the zeroth order contribution to A_{LL} has to be a constant and the first and the second orders should behave as $\epsilon \sim 1/\sqrt{s_{\gamma p}}$ and $\epsilon^2 \sim 1/s_{\gamma p}$ as functions of the energy of hard γp-scattering. The reasons why it is not true in our calculations are as follows. We start our consideration with the amplitudes of pure pomeron exchanges. We see from (18), (20), (21), (22) that, with the exception of the one pomeron exchange amplitudes, all the quark-proton amplitudes at $\Delta_T = 0$ depend on λ_P which is a linear function of $\ln(s/s_0)$ in accordance with (19). More over as $\alpha_P(0) > 1$ (see Table 1) the factors $(s/s_0)^{n\alpha_P(0)-n}$ in (18) are energy dependent and they are different for different n, hence an interference between amplitudes of m and n pomeron exchanges ($m \neq n$) depends on s. As the signature factor $\eta_P(0) \approx i$ for pomeron exchange, then the product $i^{n-1}[\eta_P(0)]^n \approx i(-1)^{n+1}$ (see (12) and (13)) changes its sign which leads to destructive interference between exchanges with odd and even numbers of pomerons. As a result of such an interference A_{LL} can change a sign as a function of the collision energy which can lead to irregular behaviour of $\ln|A_{LL}|$ versus $\ln s_{\gamma p}$ (see, for example, Fig. 10). Such an interference becomes especially important when we add nonspectator graph contributions. We show in all the figures presented in this section a behaviour of the absolute value of A_{LL} smoothed near points in which $A_{LL} = 0$ and hence $\ln|A_{LL}|$ tends to $-\infty$. If we put $d\alpha_a(t)/dt = 0$ at $t = 0$ in (19) for all reggeons under consideration and $\alpha_P(0) = 1$, then the zeroth order contribution to A_{LL} vanishes since B_4 and A_4 become independent of the collision energy (and hence of z) and therefore the integral in the numerator in (32) vanishes. The dependence of the first and

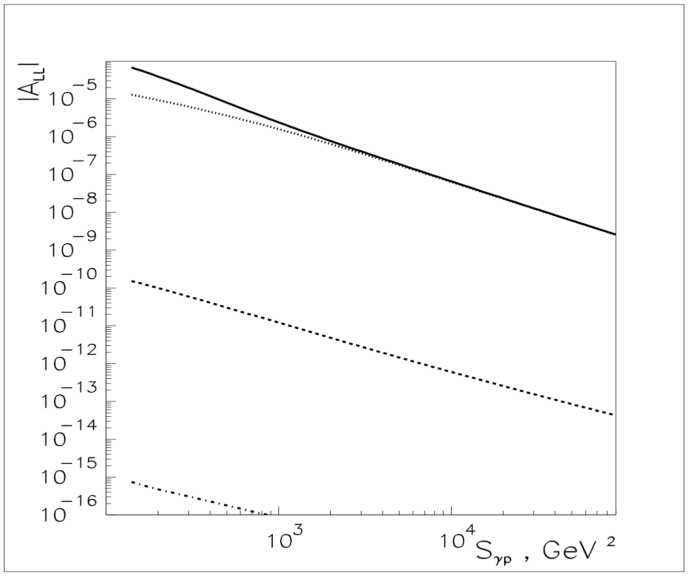


Fig. 5: Dependence of longitudinal spin-spin asymmetry on $s_{\gamma p}$. Dash-dotted and dashed curves show pure pomeron (zeroth order in $\epsilon = \sqrt{s_0/s_{\gamma p}}$) and interference of secondary reggeons with pomeron ($\sim \epsilon^1$) contributions to the absolute value of A_{LL}, respectively. Dotted curve shows contribution $\sim \epsilon^2$ to $|A_{LL}|$ and solid line represents "total asymmetry" (no decomposition into a power series in ϵ). For all curves $M_X = 10$ GeV/c^2, $Q^2 = 10$ (GeV/c)^2, $\Delta_T = 0$, $k_T \geq k_{min} = 0.2$ GeV/c.

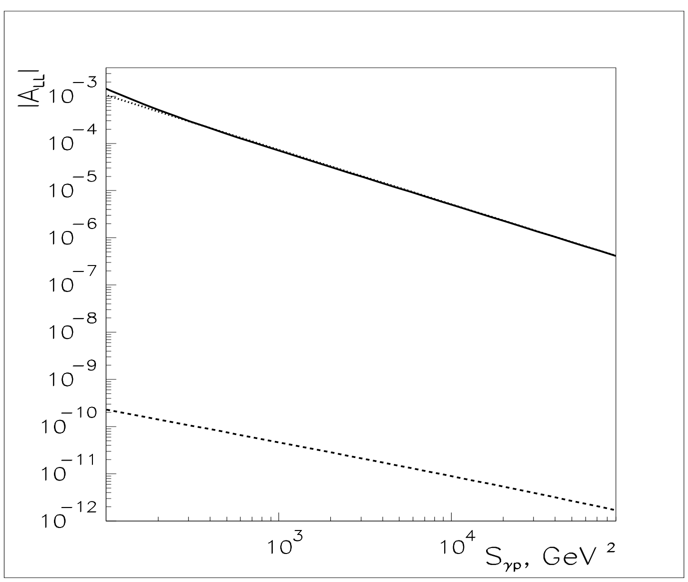


Fig. 6: Spectator graph contributions to longitudinal spin-spin asymmetry. Curves are the same as in Fig. 5 but $\alpha_P(0) = 1$, $d\alpha_a(t)/dt = 0$ at $t = 0$ for all reggeons $a = P,\ \rho,\ f,\ A_2,\ \omega$. Pure pomeron contribution to A_{LL} is equal to zero. For all curves $M_X = 5$ GeV/c^2, $Q^2 = 10$ (GeV/c)^2, $\Delta_T = 0$, $k_T \geq k_{min} = 0.2$ GeV/c.

second order contributions to the longitudinal spin-spin asymmetry becomes linear with a high accuracy if we plot $\ln|A_{LL}|$ versus $\ln(s_{\gamma p})$ and corresponds approximately to the behaviour $\sim s_{\gamma p}^{-1/2}$ and $\sim s_{\gamma p}^{-1}$, respectively. This statement is illustrated with the curves presented in Fig. 6. If we take into account the dependence of all λ_a on $s_{\gamma p}$ and the interference of contributions of different numbers of reggeon exchanges the behaviour of A_{LL} becomes more involved. We can see this comparing the solid, dashed and dotted curves in Fig. 5 with the curves of the same kind in Fig. 6.

Contributions of exchanges with different reggeons to the total amplitude of diffractive photoproduction can interfere, so the total result could in principle be much smaller than the asymmetry for the calculation of which exchanges with one reggeon c ($c = \rho,\ f,\ \omega,\ A_2$) and the pomeron have been taken into account. In this case a small value of A_{LL} is very sensitive to the values of the phenomenological parameters presented in Table 1. Any change of the parameters would destroy the cancellation of the different reggeon contributions and change the value of A_{LL} crucially. The dotted curve in Fig. 7 shows the second order contributions ($\sim \epsilon^2$) to A_{LL} due to exchanges with the pomeron and A_2-reggeon. For the calculation of the dashed curve we have taken into account pomeron and ρ-reggeon exchanges. We see also from Fig. 7 that contributions of the ω-meson trajectory (dash-dotted curve) and the f-trajectory (bold line) with some number of pomeron exchanges are less than the A_2- and ρ-reggeon exchange contributions. We are to compare these curves with the solid line representing contributions of all reggeons discussed above ($P,\ \rho,\ A_2,\ \omega,\ f$). We see that A_{LL} corresponding to the solid line is not much smaller than the longitudinal spin-spin asymmetries shown with other curves. Hence there is no destructive interference discussed above. This conclusion remains true

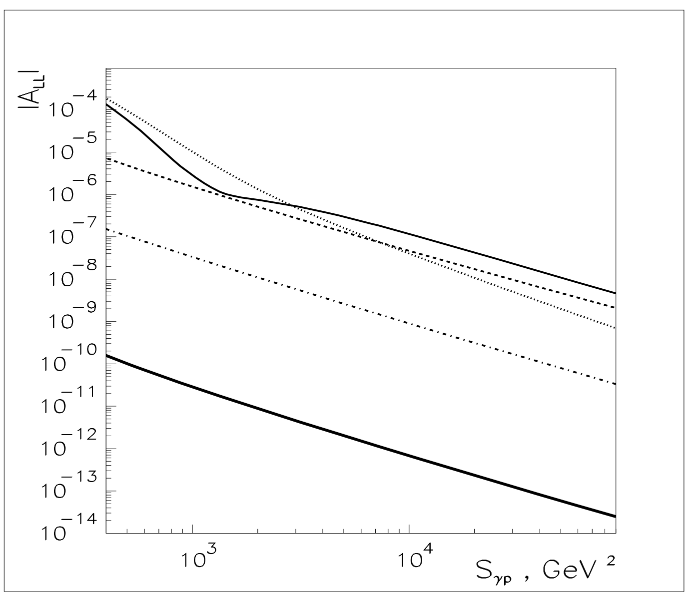


Fig. 7: Second order contributions of different reggeons to A_{LL}. Dashed ($P + \rho$), dotted ($P + A_2$), dash-dotted ($P + \omega$), bold ($P + f$) and solid ($P+\rho+A_2+\omega+f$) curves are calculated with $M_X = 20$ GeV/c^2, $Q^2 = 10$ (GeV/c)^2, $\Delta_T = 0$, $k_T \geq k_{min} = 0.2$ GeV/c for spectator graphs only.

if we add nonspectator graph contributions. So we see from a comparison of the curves presented in Fig. 7 that our estimate of A_{LL} does not depend crucially on values of the parameters found in [11], [12].

As discussed above the spectator graph contributions to A_{LL} do not depend on Q^2. Figure 8 shows that A_{LL} is not very sensitive to the value of Q^2 for $s_{\gamma p} > 400$ GeV^2 even if we add the nonspectator graph contributions to the spectator ones. We see that the difference is less than 4% for 4 (GeV/c)$^2 \leq Q^2 \leq 100$ (GeV/c)^2. In contrast to the practical independence on Q^2 at $s_{\gamma p} > 400$ GeV^2 the longitudinal spin-spin asymmetry is very sensitive to the value of the total mass M_X of hadrons produced in the hard

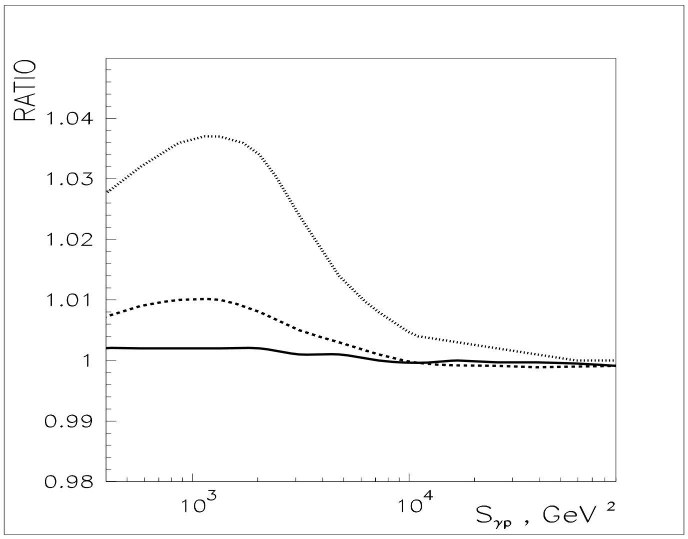


Fig. 8: Dependence of longitudinal spin-spin asymmetry on Q^2. Solid, dashed, dotted curves show ratios of spin-spin asymmetries calculated for $Q^2 = 10$, 20, 100 (GeV/c)^2 to A_{LL} obtained at $Q^2 = 4$ (GeV/c)^2. All curves are calculated with $M_X = 20$ GeV/c^2, $\Delta_T = 0$, $k_T \geq k_{min} = 0.2$ GeV/c.

γp-collision as we can see from Fig. 9a. Indeed, when M_X changes from 3 GeV/c^2 to 40 GeV/c^2, then A_{LL} increases by more than an order of magnitude. The explanation is the following. The dominant contribution of secondary reggeon trajectories to the longitudinal spin-spin asymmetry originates from low energy collisions of the quark and antiquark off the proton. The square of the center-of-mass energy of quark-proton scattering is $zs_{\gamma p}$ where at $\Delta_T = 0$ z is a root of the equation

$$z(1-z) = \frac{k_T^2 + \mu_q^2}{M_X^2} \tag{66}$$

which makes the argument of the δ-function in (65) equal to zero. If $k_T^2 \ll M_X^2$ and $\mu_q^2 \ll M_X^2$ we have the approximate solution

$$z \approx \frac{k_T^2 + \mu_q^2}{M_X^2} \tag{67}$$

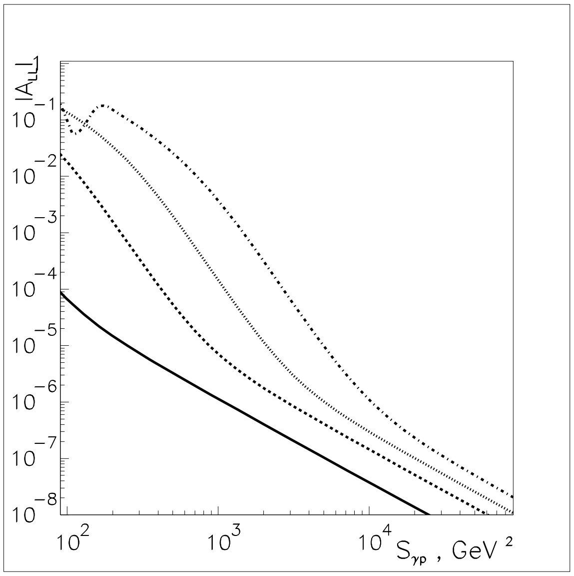


Fig. 9a: Dependence of longitudinal spin-spin asymmetry on M_X. Solid, dashed, dotted, and dash-dotted curves are calculated for M_X = 3, 10, 20, 40 GeV/c^2 all curves being computed with $Q^2 = 10$ (GeV/c)^2, $\Delta_T = 0$, $k_{min} = 0.2$ GeV/c.

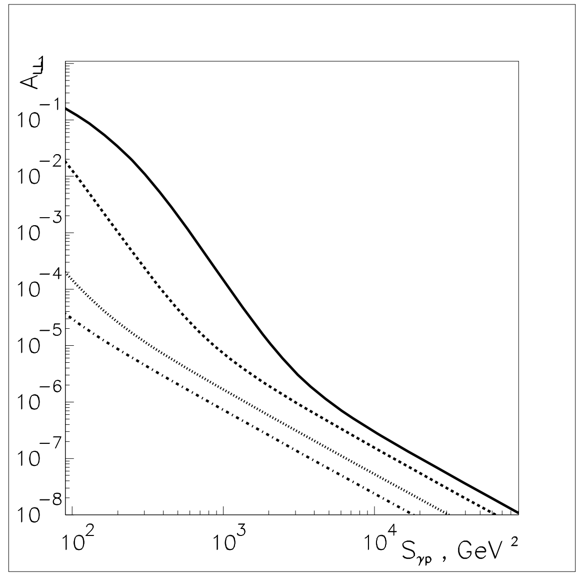


Fig. 9b: Dependence of longitudinal spin-spin asymmetry on the minimal value of k_T. Solid, dashed, dotted, and dash-dotted curves calculated for $k_{min} = 0.2$, 1, 3, 5 GeV/c. All curves are computed with $Q^2 = 10$ (GeV/c)^2, $\Delta_T = 0$, $M_X = 20$ GeV/c^2.

which shows that the greater is M_X^2 the smaller is z other things being equal. But contributions of secondary reggeon exchanges to A_{LL} increase with a decrease of the collision energy. When $1-z$ is small we have instead of (67)

$$1 - z \approx \frac{k_T^2 + \mu_q^2}{M_X^2} \; . \tag{68}$$

But $(1-z)s_{\gamma p}$ is just the square of the center-of-mass energy of antiquark-proton scattering which decreases with an increase of M_X^2. We conclude that the contribution of secondary Regge trajectories to A_{LL} increases with an increase of M_X^2 for the antiquark-proton collision too. The minimal value of z corresponds to $k_T = 0$. Putting the mass of the constituent quark $\mu_q = 350$ MeV/c^2 and $M_X = 10$ GeV/c^2 in (66) we have $z_{min} = \mu_q^2/M_X^2 \approx 0.001$. This means that for $s_{\gamma p} \approx 10^3$ GeV^2 we have for the square of the quark-proton collision energy the relation $z_{min}s_{\gamma p} \approx 1$ GeV^2. Hence for $s_{\gamma p} \ll 10^3$ GeV^2 we are out of the applicability of the Regge phenomenology used in the present paper. Strictly speaking this is true if the dominant contribution to the integral in (65) comes from those values of z for which $z(1-z)$ is close to z_{min}. Let us consider integral (65) with the lower and upper limits of z-integration z_0 and $1-z_0$, respectively. Using this auxiliary integral we can study which region of z gives the dominant contribution to integral (65). The calculations with parameters $Q^2 = 10$ (GeV/c)^2, $M_X = 10$ GeV/c^2, $s_{\gamma p} = 2000$ GeV^2 show that A_{LL} lost more than 50% of its value if $z_0 = z_{1/2} = 9 \cdot z_{min}$

and more than 70% for $z_0 = 20 \cdot z_{min}$ where $z_{min} \approx 0.001$. This is a typical example of the behaviour of integral (65) as a function of z_0. We see from the consideration of the auxiliary integral that most of the contribution to A_{LL} comes from z close to z_{min} or $1-z$ close to z_{min}. We conclude that a boundary value of $s_{\gamma p}$ for an applicability of our approach is $s_{\gamma p} \sim s_0/z_{1/2} \approx 100$ GeV^2 since $s_0 \sim 1$ GeV^2.

The main aim of our discussion is to find kinematical conditions where the contributions to A_{LL} of the soft processes are suppressed. For this case the hard process contributions dominate and we can reliably predict the longitudinal spin-spin asymmetry within the framework of perturbative QCD. As we can conclude from the considerations of Fig. 9a we are to decrease the mass of the hadronic system to decrease the soft process contribution to A_{LL}. Another possibility to suppress this contribution follows immediately from formula (66). When we select events with $k_T \geq k_{min}$, then

$$z_{min} \approx \frac{k_{min}^2 + \mu_q^2}{M_X^2} \,. \tag{69}$$

Hence for relatively large k_{min} the minimal value of the quark-proton collision energy in the center-of-mass system ($\sqrt{z_{min} s_{\gamma p}}$) can be much greater than $\sqrt{s_0}$ (we assume that $s_{\gamma p} \gg s_0$). As a result the contributions of secondary Regge trajectories to A_{LL} are suppressed. As the maximal value of z is equal to $1-z_{min}$ the antiquark-proton scattering energy is greater than $\sqrt{(1-z_{max})s_{\gamma p}} = \sqrt{z_{min} s_{\gamma p}}$ and the soft process contribution to A_{LL} in antiquark-proton scattering is suppressed too. Figure 9b illustrates this possibility to decrease the soft process contribution to the asymmetry. We see that increasing k_{min} from 0.2 GeV/c to 1 GeV/c we decrease A_{LL} by more than an order of magnitude at $s_{\gamma p} \sim 10^2$ GeV^2 where the longitudinal spin-spin asymmetry has the largest values. At higher energies the suppression of the reggeon exchange contributions to A_{LL} is significant too when we apply the cut $k_T \geq k_{min} \geq 1$ GeV/c. The last kinematical condition means that we consider experimental events with two jets having the difference between their transverse momenta greater than $2k_{min}$. If we can reliably measure longitudinal hadron momenta we can exclude events with a very low value of $z(1-z)$ (when $z s_{\gamma p} \sim s_0$ or $(1-z)s_{\gamma p} \sim s_0$). This way suppresses essentially the contributions to A_{LL} of secondary reggeon trajectories too. We conclude from the discussion of Figs. 9a and 9b that decreasing M_X or increasing k_{min} we increase z_{min} (and decrease $z_{max} = 1 - z_{min}$ too). As a result we suppress the contributions of $\rho,\ \omega,\ f,\ A_2$ to A_{LL}.

The results of the calculations of A_{LL} at $\Delta_T > 0$ are presented in Fig. 10. We see from the comparison of the curves in Fig. 10a that the pure pomeron contribution to A_{LL} increases significantly with an increase of Δ_T and at $\Delta_T = 1.5$ GeV/c takes on a value $\sim 10^{-6}$ which is much greater than at $\Delta_T = 0$. It is easy to understand the increase of the pomeron exchange contributions to A_{LL} for the spectator graphs. We see from formulæ (18), (20), (21), (22) that $A_5 = 0$ even at $\Delta_T \neq 0$ (A_5 is defined in (17)). Hence the longitudinal spin-spin correlations $(\vec{\sigma}_1 \cdot \vec{l})(\vec{\sigma}_2 \cdot \vec{l})$ and $(\vec{\sigma}_1 \cdot \vec{l})(\vec{\sigma}_3 \cdot \vec{l})$ in the product $F_q^+(\vec{\Delta}_T) F_q(\vec{\Delta}_T)$ in (65) which give A_{LL} can arise in the product of the amplitudes A_3 and A_4 as

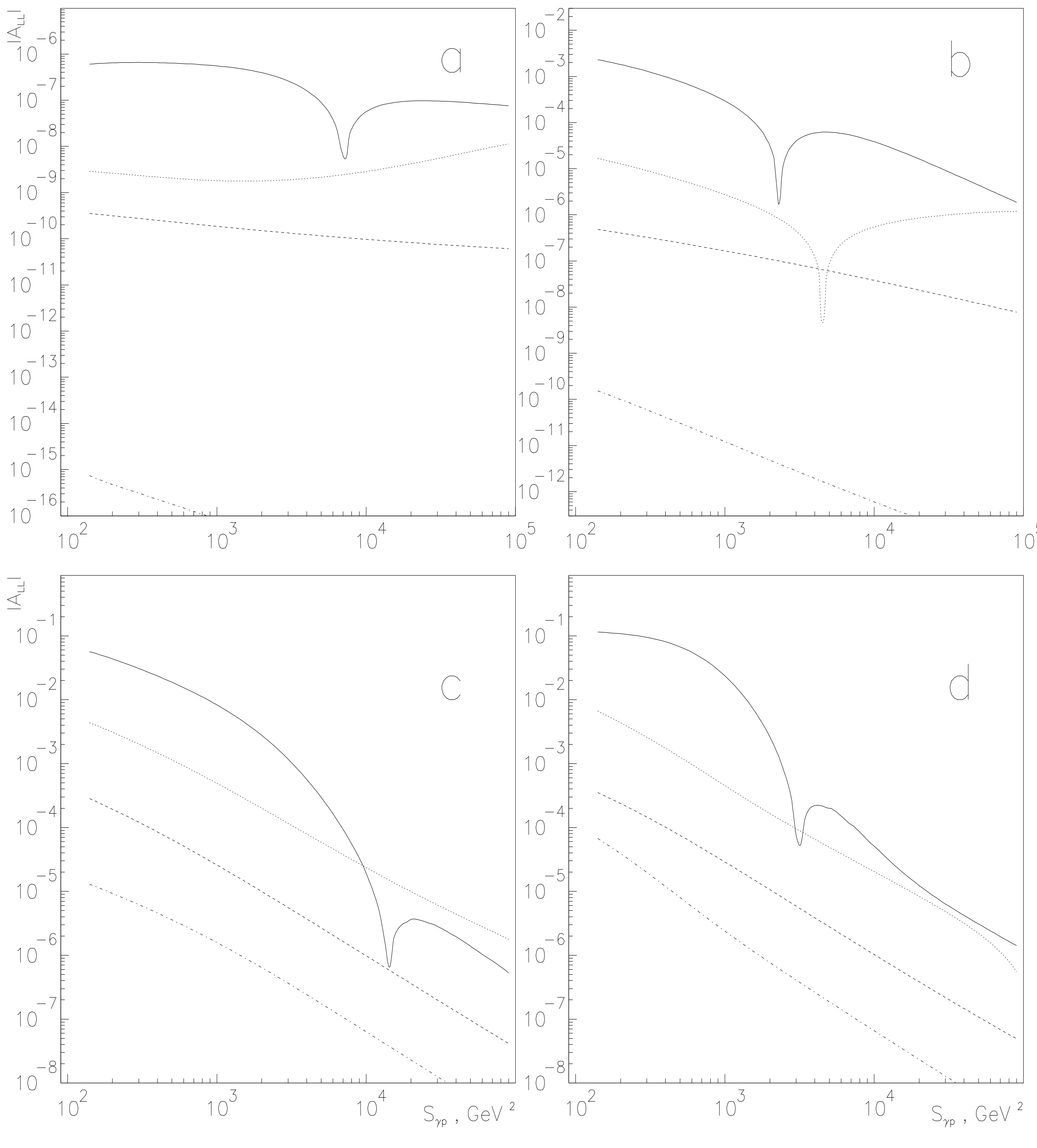


Fig. 10: Dependence of longitudinal spin-spin asymmetry on Δ_T. Figs. 10a and 10b show pure pomeron (zeroth order in $\epsilon = \sqrt{s_0/s_{\gamma p}}$) contribution to $|A_{LL}|$ and interference of contributions of secondary reggeons with pomeron ($\sim \epsilon^1$), respectively. Figs. 10c and 10d show second order contribution to $|A_{LL}|$ and "total asymmetry" (no decomposition into a power series in ϵ). Dash-dotted, dashed, dotted and solid curves are calculated for $\Delta_T = 0,\ 0.5,\ 1.0,\ 1.5$ GeV/c, respectively. All curves are smoothed near points where $A_{LL} = 0$. For all curves $M_X = 10$ GeV/c^2, $Q^2 = 10$ (GeV/c)^2, $k_{min} = 0.2$ GeV/c.

$$(\vec{\sigma}_1 \cdot \vec{m})(\vec{\sigma}_j \cdot \vec{m})(\vec{\sigma}_1 \cdot \vec{n})(\vec{\sigma}_j \cdot \vec{n}) = -(\vec{\sigma}_1 \cdot \vec{l})(\vec{\sigma}_j \cdot \vec{l}) \tag{70}$$

where $j = 2$ in (70) for the quark-proton collision and $j = 3$ for antiquark-proton scattering. But A_3 and A_4 at $\Delta_T = 0$ are equal to zero for the one and two pomeron exchange amplitudes and the three pomeron exchange amplitudes A_3 and A_4 are very small compared with A_1 (they contain the small factor $p_y^6/(243\lambda_P^5)$ as one can see from (22)). The amplitudes $A_3^{(n)}$ of the one ($n = 1$) and two ($n = 2$) pomeron exchanges are proportional to Δ_T^2 and increase when Δ_T increases from zero to some value. It is the increase of A_3 which causes the increase of the pure pomeron contribution to A_{LL} shown in Fig. 10a. We see also from a comparison of the curves presented in Fig. 10b that the first order contribution to A_{LL} at $\Delta_T > 0.5$ GeV/c is much greater than at $\Delta_T = 0$. It is proportional to the sum of products of the amplitudes of pure pomeron exchanges ($\sim \epsilon^0$) by the amplitudes ($\sim \epsilon^1$) of exchanges with one secondary reggeon and one or two pomerons. The first order contribution to A_{LL} increases with Δ_T when the absolute value of the pure pomeron exchange amplitudes increase. A comparison of the curves calculated at different Δ_T in Figs. 10c and 10d shows an increase of the second order contributions to A_{LL} and "the total asymmetry" with Δ_T.

Up to now we have considered Regge trajectories containing resonances with natural parity $\pi = \sigma = (-1)^J$ where π and J denote parity and total spin of a resonance. It follows from the parity conservation and the T-invariance of the strong interaction that the dependence on the spin variables of the qqB- and NNB-vertices for a reggeon B with natural parity has the form given by (3) and (5) (see, for instance, [10], [18], [19]). For the Regge trajectories under discussion the one reggeon exchange amplitudes A_4 and A_5 of qN- and $\bar{q}N$-scattering in (17) are equal to zero hence the longitudinal spin-spin correlation in the cross section $(\vec{\sigma}_1 \cdot \vec{l})(\vec{\sigma}_j \cdot \vec{l})$ is absent ($j = 2$ (or $j = 3$) for scattering of a quark (or an antiquark) on the proton). The nonzero amplitudes A_4 and A_5 can appear for many reggeon exchanges only and their contribution to A_{LL} becomes nonzero. For unnatural parity trajectories (with $\sigma\pi = -1$) there are two alternatives. The NNB- and qqB-vertices look like either

$$B = \Delta_T B_x(\Delta_T^2)(\vec{\sigma}_1 \cdot \vec{m}) \ , \quad b = \Delta_T b_x(\vec{\sigma}_j \cdot \vec{m}) \tag{71}$$

or

$$B = B_z(\Delta_T^2)(\vec{\sigma}_1 \cdot \vec{l}) \ , \quad b = b_z(\vec{\sigma}_j \cdot \vec{l}) \tag{72}$$

which is also a consequence of the parity conservation and the time reverse invariance [10], [18]. It follows from the G-parity conservation (see relations for the helicity amplitudes in [18], [19]) that formula (71) is applicable for π-reggeon exchange ($T^G = 1^-, \ J^\pi = 0^-$) and for exchange with the $A_1(1260)$-meson ($T^G = 1^-, \ J^\pi = 1^+$) the vertices are described by relation (72). For the pion trajectory we have $\alpha_\pi(0) \approx 0$. The intercept of the A_1-trajectory is not well established but the region for it is the following: $-0.25 \leq \alpha_{A_1}(0) \leq 0$ [10], [20]. For the π and A_1 Regge trajectories the one reggeon exchange amplitudes are at least quantities $\sim \epsilon^2 \sim s_0/s$ where s is the square of the center-of-mass energy of colliding particles. Since the π- and A_1-reggeon exchange amplitudes decrease with the collision energy more rapidly ($\sim s^{-n}, \ n \geq 1$) than the amplitudes

of f, ρ, ω, A_2 ($\sim 1/\sqrt{s}$) and pomeron exchanges, hence their contributions to A_{LL} are suppressed compared with the natural parity reggeon contributions at $s \to \infty$. But they contribute to the longitudinal spin-spin asymmetry even in the one reggeon exchange approximation. Indeed, the pomeron-pion interference term in the cross section gives the longitudinal spin-spin correlation $(\vec{\sigma}_1 \cdot \vec{l})(\vec{\sigma}_j \cdot \vec{l})$ at nonzero Δ_T due to (70). Recall that the one pomeron exchange amplitude contains the term $A_3(\vec{\sigma}_1 \cdot \vec{n})(\vec{\sigma}_j \cdot \vec{n})$ (see (20)) and the one π-reggeon exchange amplitude has the term $A_4(\vec{\sigma}_1 \cdot \vec{m})(\vec{\sigma}_j \cdot \vec{m})$ according to (71). The correlation term $(\vec{\sigma}_1 \cdot \vec{l})(\vec{\sigma}_j \cdot \vec{l})$ is equal to zero if the spin-dependent vertex of the pomeron p_y or P_y in (3), (5) is zero. On the other hand for exchange with $A_1(1260)$ the amplitude A_5 is nonzero as this follows from (72). Hence the longitudinal spin-spin term $(\vec{\sigma}_1 \cdot \vec{l})(\vec{\sigma}_j \cdot \vec{l})$ exists and produces nonzero A_{LL} even if the spin-dependent part of the pomeron vertex is equal to zero.

For the numerical calculations we have used the expression [11]

$$A_4^{(\pi)} = \frac{G^2_{\pi NN}}{16\pi m_N E_0} \frac{0.6t}{(t-m_\pi^2)} \frac{s}{s-m_N^2-m_q^2} \Big(\frac{s}{s_0}\Big)^{\alpha_\pi(m_\pi^2)-1}$$
$$\exp\Big\{-\Big[\frac{r_\pi^2}{2} + \alpha_\pi^{'}(m_\pi^2)\Big(\ln\Big(\frac{s}{s_0}\Big) - i\frac{\pi}{2}\Big)\Big](m_\pi^2 - t)\Big\}(\vec{\tau}_1 \cdot \vec{\tau}_j) \tag{73}$$

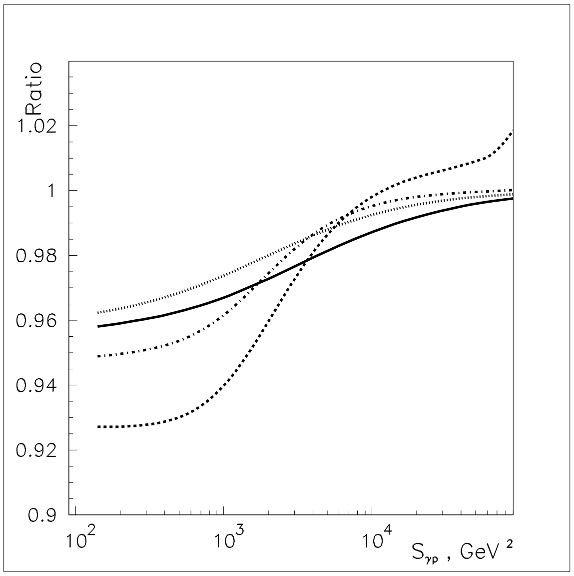
for the one π-reggeon exchange amplitude where $G^2_{\pi NN}/(4\pi) = 14.6$, $G_{\pi NN}$ is the pion-nucleon constant, $t \approx -\Delta_T^2$, $r_\pi^2 = 3$ (GeV/c)$^{-2}$, $\alpha_\pi^{'}(m_\pi^2) = 1$ (GeV/c)$^{-2}$ [11], m_π and m_N denote the pion and nucleon masses, respectively. The additional factor 0.6 in (73) compared with the NN-scattering amplitude is due to the relation between the qqB- and NNB-vertices for $B = \pi$ which looks like $b_x = 0.6B_x$. This relation is in agreement with the second relation (37) since π has isospin $T = 1$. We have written $r_\pi^2/2$ in (73) instead of r_π^2 as we consider the $qq\pi$-vertex as the vertex of reggeon emission by a point-like quark. As usual $j = 2$ in (73) corresponds to the quark-proton collision and $j = 3$ does to antiquark-proton scattering. We parameterize the $A_1(1260)$-reggeon exchange amplitude as in [10]

$$A_5^{(A_1)} = -\frac{1}{16\pi s} A_{1z}(\vec{\sigma}_1 \cdot \vec{l}) a_{1z}(\vec{\sigma}_j \cdot \vec{l})[1 - \exp\{-i\pi\alpha_{A_1}(t)\}]$$
$$\Gamma(1-\alpha_{A_1}(t))(\alpha_{A_1}^{'}(0)s)^{\alpha_{A_1}(t)}(\vec{\tau}_1 \cdot \vec{\tau}_j)\ . \tag{74}$$

In (74) $\Gamma(x)$ denotes the Euler gamma function. The vertex parameter A_{1z} in (74) is put equal to $6.2/\sqrt{2}$ in accordance with [10], $a_{1z} = 0.6A_{1z}$ in agreement with (37). The results of our estimate of the $A_1(1260)$-exchange contribution to the longitudinal spin-spin asymmetry are shown in Fig. 11a for the spectator graphs only. The curves show a behaviour of the ratios of A_{LL} calculated with and without contributions of one A_1-reggeon exchange, exchanges with P, ρ, f, A_2 and ω reggeons being taken into account for all the curves in Fig. 11a. We have calculated ratios under discussion for $\alpha_{A_1}(0) = 0$ and $\alpha_{A_1}(0) = -0.2$ finding the slope of the linear A_1-trajectory $\alpha_{A_1}^{'}(0)$ from the relation

$$1 = \alpha_{A_1}(0) + \alpha_{A_1}^{'}(0)m_{A_1}^2$$

with the mass of the A_1-meson m_{A_1} equal to 1.23 GeV/c^2. We see from Fig. 11a that the relative contribution of A_1 at $100 < s_{\gamma p} < 9 \cdot 10^4$ GeV^2 is less than 8%. Though the



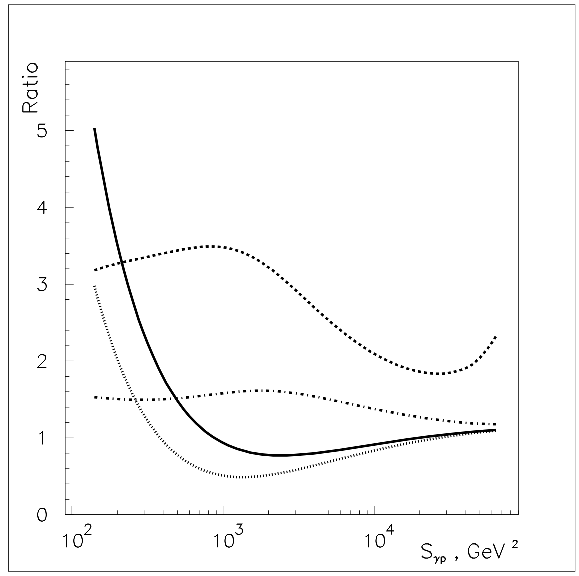


Fig. 11a: Relative contribution of $A_1(1260)$-reggeon to A_{LL}. All curves show ratios of A_{LL} calculated with and without one A_1-reggeon exchange contribution. Solid and dashed curves are calculated for $\alpha_{A_1}(0) = 0$ at $\Delta_T = 0.5$ GeV/c and 1 GeV/c, respectively. Dotted and dash-dotted curves are obtained for $\alpha_{A_1}(0) = -0.2$ at $\Delta_T = 0.5$ GeV/c and 1 GeV/c, respectively. Spectator graphs have been considered only. For all curves contributions of $P,\ f,\ \rho,\ \omega,\ A_2$ are taken into account for $Q^2 = 10$ (GeV/c)^2, $M_X = 10$ GeV/c^2, $k_{min} = 0.2$ GeV/c.

Fig. 11b: Relative contribution of π-reggeon to longitudinal spin-spin asymmetry. Curves show ratios of A_{LL} calculated with and without π-reggeon exchange contribution, spectator and non-spectator graphs being taken into account. Solid, dotted curves are calculated at $\Delta_T = 0.5$ GeV/c and dashed, dash-dotted curves are obtained for $\Delta_T = 1.0$ GeV/c. Computing solid and dashed curves we take into account π and $\pi + P$ exchange amplitudes. In obtaining dotted and dash-dotted curves exchanges with π, $\pi + P$ and $\pi + P + P$ are included into calculation. Parameters $Q^2,\ M_X,\ k_{min}$ are the same as in Fig. 11a and contributions of $P,\ f,\ \rho,\ \omega,\ A_2$ are taken into account.

amplitude of A_1-exchange has the spin-spin term $(\vec{\sigma}_1 \cdot \vec{l})(\vec{\sigma}_j \cdot \vec{l})$ most suitable to produce A_{LL} and interferes with the spin-independent part of the pomeron exchange amplitude (which is large) the contribution of the A_1-trajectory to A_{LL} is small due to two factors: $-0.2 \le \alpha_{A_1}(0) \le 0$ and the negative signature σ of A_1. Due to the unequality $\alpha_{A_1}(0) \le 0$ the A_1 exchange amplitude contains the factor $(s/s_0)^{\alpha_{A_1}(t)-1} \le s_0/s$ which is small at large energies. The factor $1 + \sigma \exp\{-i\pi\alpha_{A_1}(t)\}$ in (74) suppresses the contribution of the $A_1(1260)$-reggeon since $\sigma = -1$ and $\alpha_{A_1}(t)$ is small (this factor is especially small for $\alpha_{A_1}(0) = 0$). Pion exchange gives more appreciable contribution to A_{LL} than A_1 as we see from Fig. 11b. For the pion having $\sigma = 1$ the factor $1 + \sigma \exp\{-i\pi\alpha_{A_1}(t)\}$ is of the order of unity besides the constant $G_{\pi NN}$ in (73) is large. This two facts explain

why the contribution of π to A_{LL} is greater than the A_1-reggeon exchange contribution. Since π-reggeon exchange gives the contribution to A_{LL} of the same order of magnitude as the ρ-, f-, A_2-, ω-contributions for the energies achieved at HERA we have to take into account in our numerical calculations not only the pole term but the branch cats too. The intercept $\alpha_\pi(0) \approx 0$ hence $(s/s_0)^{\alpha_\pi(0)-1} \sim s^{-1} \sim \epsilon^2$ therefore we are to include into consideration π, πP and πPP exchanges only. This has been done in obtaining curves presented in Fig. 11b. They are calculated in two approximations. In the former approximation we take into account the amplitudes of pion and pion + pomeron exchanges. In the latter approximation we add to these amplitudes the amplitude of $\pi + P + P$ exchanges. The applied formulæ are presented in the Appendix. Since the contribution of the A_1 Regge trajectory is small we restrict our consideration with the pole amplitude given by relations (17) and (74).

Let us analyse the results presented in Figs. 5, 8, 9, 10, 11 from the point of view of the main idea of the present paper. We remind that we try to find kinematical conditions to suppress the reggeon contributions to the longitudinal spin-spin asymmetry making them much smaller than perturbative QCD contributions to A_{LL}. The latter contributions can be more reliably predicted theoretically and compared with experimental data than the contributions of the soft processes considered in the present paper. We see from Fig. 10d that $|A_{LL}| < 10^{-5}$ at $s_{\gamma p} \geq 10^4$ GeV^2 and $\Delta_T < 1$ GeV/c. It follows from Figs. 11a and 11b that even if we include in the calculation the contributions of the π- and $A_1(1260)$-reggeons $|A_{LL}|$ will be smaller than 10^{-4}. We see from Figs. 9a and 9b that we reduce a value of A_{LL} decreasing M_X or increasing the minimal value of k_T (the demand $k_T > k_{min}$ means that one selects events with a difference between transverse momenta of quark jets greater than $2k_{min}$). In these cases we increase a minimal fraction of the virtual photon momentum z carried by a quark/antiquark. This follows immediately from formula (69). The quantity of z_{min} is the most important variable which can be changed experimentally to suppress the reggeon contributions to A_{LL}. Values of $|A_{LL}| < 10^{-4}$ cannot presumably be measured by modern experimental technique since one needs much more than 10^8 events to have statistical errors of the γp cross sections much less than 10^{-4}. If $|A_{LL}|$ predicted within the framework of perturbative QCD is greater than 10^{-4} we do not need any additional kinematical cuts at $s_{\gamma p} \geq 10^4$ GeV^2 and $\Delta_T < 1$ GeV/c to suppress the reggeon contributions to the longitudinal spin-spin asymmetry. We have argued above that our approach is applicable at $s_{\gamma p} \geq 10^3$ GeV^2. We see from the presented numerical results that A_{LL} at $s_{\gamma p} = 10^3$ GeV^2 can be $\sim 10^{-2}$. For the region $10^3 \leq s_{\gamma p} \leq 10^4$ GeV^2 we have to take into account the contributions to A_{LL} not only of P, f, ρ, ω, A_2 Regge trajectories but A_1- and π-reggeons as well. As we have told the most important parameter influencing a value of the longitudinal spin-spin asymmetry is z_{min}. Figure 12 shows the dependence of A_{LL} on the lower limit (z_{min}) in the integral over z in (65) for $s_{\gamma p} = 10^3$ GeV^2 the upper limit in the integral being put equal to $z_{max} = 1 - z_{min}$. The contributions both of P, f, ρ, ω, A_2 and π, A_1 reggeons are taken into account in the calculations. We see that $|A_{LL}|$ is less than 10^{-4} for $\Delta_T = 0.5$ GeV/c. The absolute value of the longitudinal spin-spin asymmetry at $\Delta_T = 1$ GeV/c becomes less than 10^{-4} for $z_{min} > 0.025$. Remembering formula (69) we get the region $k_T^2 \geq 0.025 M_X^2$ at $s_{\gamma p} \geq 10^3$ GeV^2 and $\Delta_T \leq 1$ GeV/c in which $|A_{LL}| \leq 10^{-4}$. In this kinematical regions we can reliably compare the perturbative QCD predictions for A_{LL} with experimental data to

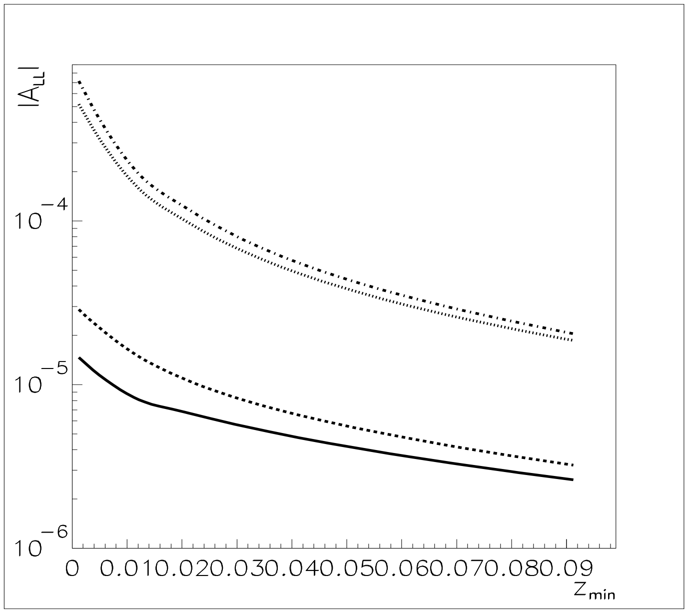


Fig. 12: Dependence of longitudinal spin-spin asymmetry on z_{min}. Dashed and solid curves show contributions $\sim \epsilon^2$ to A_{LL} and "total asymmetry" at $\Delta_T = 0.5$ GeV/c. Dotted and dash-dotted lines are the same as dashed and solid curves but for $\Delta_T = 1.0$ GeV/c. For all curves $s_{\gamma p} = 10^3$ GeV^2, $M_X = 10$ GeV/c^2, $Q^2 = 10$ (GeV/c)^2, $k_{min} = 0.2$ GeV/c.

be obtained in the nearest future at the HERA collider in scattering of the polarized electrons/positrons off the polarized protons.

6 Conclusions

We have calculated the contributions of P, $\rho(770)$, $f_2(1270)$, $A_2(1320)$, $\omega(782)$, π and $A_1(1260)$ reggeon exchanges to the longitudinal spin-spin asymmetry, A_{LL} in the diffractive hadron production in hard γp-scattering at energies accessible at HERA ($10^2 \leq s_{\gamma p} \leq 10^5$ GeV^2). Our numerical predictions are obtained with the aid of the phenomenological parameters found in [10], [11], [12] from the study of hadron-hadron scattering within the framework of the Regge theory. We restrict our consideration with the amplitudes of exchanges with one, two, and three reggeons only and decompose A_{LL} into a power series in ϵ ($\epsilon = \sqrt{s_0/s_{\gamma p}}$). We study terms $\sim \epsilon^0$, ϵ^1, ϵ^2 and the total longitudinal spin-spin asymmetry (no decomposition into power series in ϵ) . It is shown that the dominant contributions to A_{LL} at $10^2 \leq s_{\gamma p} \leq 10^5$ GeV^2 are the second order contributions ($\sim \epsilon^2$) in spite of a validity of the inequality $\epsilon^0 \gg \epsilon^1 \gg \epsilon^2$. The pure pomeron exchange contributions ($\sim \epsilon^0$) to A_{LL} are very small ($|A_{LL}| < 10^{-6}$ at $\Delta_T \leq 1.5$ GeV/c, $M_X \leq 10$ GeV/c^2) at energies achieved at the collider HERA. One can neglect them with respect to the other contributions. Exchanges with one secondary reggeon and some number of the pomerons ($\sim \epsilon^1$) are comparable with the second order contributions to A_{LL} for momenta transferred to the proton $\Delta_T \sim 1$ GeV/c and are

negligible at $\Delta_T = 0$. The dominant contribution to the numerator in the formula for A_{LL} comes from those z (the heavy photon momentum fraction carried by a quark) for which z (or $1 - z$) is close to its lower limit z_{min}. In this z-region, the center-of-mass energy of quark-proton (antiquark-proton) scattering can be rather low (~ 1 GeV) and π-reggeon exchange becomes very important though the intercept of the pion Regge trajectory ($\alpha_\pi(0) \approx 0$) is smaller than the intersepts of $P,\ \rho,\ f_2,\ A_2,\ \omega$ trajectories. The A_1-reggeon has the suitable spin structure of the qqA_1-vertex to produce the longitudinal spin-spin asymmetry even in the one reggeon exchange approximation when the spin-dependent pomeron vertex is equal to zero. Nevertheless the relative contribution of the $A_1(1260)$-trajectory to A_{LL} is less than 10% since A_1 has the negative signature and the small coupling constant.

The main purpose of the present paper is to find kinematical conditions where $|A_{LL}| < 10^{-4}$ (the lowest limit for A_{LL} which can presumably be measured by modern experimental technique). It is shown that $|A_{LL}| < 10^{-4}$ at $s_{\gamma p} \geq 10^3$ GeV^2, $\Delta_T \leq 0.5$ GeV/c, and the mass of produced hadrons, $M_X \leq 10$ GeV/c^2. The longitudinal spin-spin asymmetry at $Q^2 \geq 4$ (GeV/c)^2 is practically insensitive to Q^2 hence we do not need any additional cuts for Q^2. For $\Delta_T \leq 1$ GeV/c (other conditions are as before) we need some cuts to reduce A_{LL}. For this aim we have to make z_{min} higher (to increase the quark-proton and antiquark-proton collision energy) than 0.03. Selecting experimental events with $k_T^2 \geq 0.03 M_X^2$ we make $|A_{LL}|$ smaller than 10^{-4}. If the perturbative QCD contribution to A_{LL} is greater than 10^{-4}, then making the cuts to suppress the soft Regge process contributions one can reliably compare the perturbative QCD predictions for A_{LL} with the experimental data which can be obtained in the future at HERA in hard scattering of the polarized electrons/positrons off the polarized protons.

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Appendix

Spectator graphs

In the Appendix we present the general formulæ for the contributions of one, two and three reggeon exchanges both for the spectator and non-spectator graphs. For the spectator graphs the amplitude of the $q\bar{q}$-pair production and its rescattering on the proton is given by the general relation (63). If isospin of a reggeon b is equal to zero, then invariant amplitudes $A_j^{(1)}$ corresponding to exchange with the reggeon b can be described by relations (18), (19), (20) where we are to substitute $b_s,\ b_y,\ B_s,\ B_y$ instead of $p_s,\ p_y,\ P_s,\ P_y$. When isospin of the reggeon b is equal to 1 we have the relations

$$
\begin{aligned}
a_1^{(1)} &= b_s B_s (\vec{\tau}_1 \cdot \vec{\tau}_j) \ , \\
a_2^{(1)} &= i\Delta_T b_y B_s (\vec{\tau}_1 \cdot \vec{\tau}_j) \ , \\
a_6^{(1)} &= i\Delta_T B_y b_s (\vec{\tau}_1 \cdot \vec{\tau}_j) \ , \\
a_3^{(1)} &= -\Delta_T^2 b_y B_y (\vec{\tau}_1 \cdot \vec{\tau}_j) \ ,
\end{aligned} \tag{75}
$$

instead of (20) where $j = 2$ in (75) for qN-scattering and $j = 3$ for the $\bar{q}N$-collision. The total one reggeon exchange amplitude is a sum over all reggeons b which can contribute to the process under discussion.

For exchange with two reggeons, say, b and h, both having isospin $T = 0$ the invariant amplitudes are given by the formulæ

$$
\begin{aligned}
A_l^{(2)} &= \sum_{b,h} \frac{f_I(b,h)}{2!} C_{sh}^{(2)} \eta_b(0)\eta_h(0)(s/s_0)^{\alpha_b(0)-\alpha_h(0)-2} \exp\{-\lambda\Delta_T^2\} a_l^{(2)}(b,h) \ , \\
a_1^{(2)}(b,h) &= \frac{i}{\lambda_1}[b_s h_s B_s H_s - (b_y h_y B_s H_s + B_y H_y b_s h_s)(\lambda\Delta_T^2 - 1)/\lambda_1 \\
&\quad + b_y h_y B_y H_y (\lambda^2 \Delta_T^4 + \lambda_1 \Delta_T^2/2 - 4\lambda\Delta_T^2 + 2)/\lambda_1^2] \ , \\
a_2^{(2)}(b,h) &= -\frac{\Delta_T}{\lambda_1^2}\{\lambda_h b_y h_s + \lambda_b h_y b_s)[B_s H_s - B_y H_y (\lambda\Delta_T^2 - 1)/\lambda_1] \\
&\quad - \frac{\lambda_2}{2\lambda_1} B_y H_y (b_y h_s - h_y b_s)\} \ , \\
a_6^{(2)}(b,h) &= -\frac{\Delta_T}{\lambda_1^2}\{\lambda_h B_y H_s + \lambda_b H_y B_s)[b_s h_s - b_y h_y (\lambda\Delta_T^2 - 1)/\lambda_1] \\
&\quad - \frac{\lambda_2}{2\lambda_1} b_y h_y (B_y H_s - H_y B_s)\} \ , \\
a_3^{(2)}(b,h) &= -\frac{i}{\lambda_1^2}[\frac{1}{2}(b_y h_s - h_y b_s)(B_y H_s - H_y B_s) + \frac{\Delta_T^2}{\lambda_1}(\lambda_h b_y h_s + \lambda_b h_y b_s) \\
&\quad (\lambda_h B_y H_s + \lambda_b H_y B_s)] \ , \\
a_4^{(2)}(b,h) &= -\frac{i}{2\lambda_1^2}(b_y h_s - h_y b_s)(B_y H_s - H_y B_s) \ .
\end{aligned} \tag{76}
$$

In (76) we have presented the nonzero amplitudes $a_j^{(2)}(b,h)$ only and applied the short notations

$$
\begin{aligned}
\lambda &= \lambda_b \lambda_h/(\lambda_b + \lambda_h) \ , \\
\lambda_1 &= \lambda_b + \lambda_h \ , \\
\lambda_2 &= \lambda_b - \lambda_h \ ,
\end{aligned} \tag{77}
$$

where λ_a for any reggeon a has been defined by (19). The qqa-vertex is given by the relation

$$
a(\Delta) = a_s + i a_y (\vec{\sigma}_2 \cdot \vec{l} \times \vec{\Delta}_T) \ , \tag{78}
$$

and the NNa-vertex is

$$
A(\Delta) = A_s(\Delta_T) + i A_y(\Delta_T)(\vec{\sigma}_1 \cdot \vec{l} \times \vec{\Delta}_T) \ . \tag{79}
$$

The isospin factor $f_I(b,h) = 1$ in (76) when the reggeons b and h have isospins $T = 0$. Formulæ (76) can be applied for nonzero isospins. When isospin of one reggeon only, say b, is equal to 1, then the isospin factor is

$$f_I(b,h) = (\vec{\tau}_1 \cdot \vec{\tau}_j) \tag{80}$$

with $j = 2$ for qN-scattering and $j = 3$ for $\bar{q}N$-scattering. As we do not consider nucleon charge exchange, then the isospin factor is equal to

$$f_I(b,h) = 2m_j \tag{81}$$

where $m_j = \pm\frac{1}{2}$ denotes the third component of quark ($j = 2$) isospin (or antiquark ($j = 3$) isospin if we consider antiquark-proton scattering). When isospins of the reggeons $b,\ h$ are equal to 1, then the factor $f_I(b,h)$ in (76) is

$$f_I(b,h) = 3\ . \tag{82}$$

For this case the amplitude $a_5^{(2)}(b,h)$ becomes nonzero

$$a_5^{(2)}(b,h) = \frac{i\Delta_T^2}{3\lambda_1^2} b_y h_y B_y H_y (\vec{\tau}_1 \cdot \vec{\tau}_j)\ . \tag{83}$$

As charge exchange of the proton is not considered here we can replace $(\vec{\tau}_1 \cdot \vec{\tau}_j)$ by $2m_j$ and hence the final formula for $a_5^{(2)}(b,h)$ becomes as follows:

$$a_5^{(2)}(b,h) = \frac{2im_j\Delta_T^2}{3\lambda_1^2} b_y h_y B_y H_y\ . \tag{84}$$

It is interesting to point out that the amplitude $A_4^{(2)}$ which contributes to the spin-spin asymmetry (even if we consider the spectator graphs only) is equal to zero identically when $b = h$. This means that the two pomeron exchange contribution to A_{LL} vanishes. We see also from (76) that $a_4^{(2)}(b,h)$ is nonzero for exchanges with two different reggeons and is equal to $a_3^{(2)}(b,h)$ at $\Delta_T = 0$. Hence for this case the sum of the amplitudes A_3 and A_4 is proportional to $(\vec{\sigma}_{1T} \cdot \vec{\sigma}_{2T})$ in accordance with (17). It follows from formula (32) for A_{LL} that the amplitude A_5 can in principle contribute to the numerator but for exchanges with two reggeon $A_5 = 0$ at $\Delta_T = 0$ what one can see from (76) and (84).

Let us consider three reggeon exchanges. In the present paper we restrict ourselves with the case when not more than two of exchanged reggeons are not the pomeron as we consider terms $\sim \epsilon^0,\ \epsilon,\ \epsilon^2$ only. Hence one reggeon among three reggeons is the pomeron. For a compact representation of the final formulæ let us introduce short notations

$$\begin{aligned}
S = b_s h_s c_s B_s H_s C_s, \quad Y = b_y h_y c_y B_y H_y C_y,\\
S_y = b_s h_s c_s B_y H_y C_y, \quad Y_s = b_y h_y c_y B_s H_s C_s,\\
a = b_y/b_s + h_y/h_s + c_y/c_s, \quad A = B_y/B_s + H_y/H_s + C_y/C_s\ ,\\
c = b_s/b_y + h_s/h_y + c_s/c_y, \quad C = B_s/B_y + H_s/H_y + C_s/C_y\ ,\\
w = b_y/(b_s\lambda_b) + h_y/(h_s\lambda_h) + c_y/(c_s\lambda_c)\ ,
\end{aligned}$$

$$W = B_y/(B_s\lambda_b) + H_y/(H_s\lambda_h) + C_y/(C_s\lambda_c) \; ,$$
$$e = \lambda_b b_s/b_y + \lambda_h h_s/h_y + \lambda_c c_s/c_y, \quad E = \lambda_b B_s/B_y + \lambda_h H_s/H_y + \lambda_c C_s/C_y \; ,$$
$$U = b_y B_y/(b_s B_s \lambda_b) + h_y H_y/(h_s H_s \lambda_h) + c_y C_y/(c_s C_s \lambda_c) \; ,$$
$$V = \lambda_b b_y B_s/(b_s B_y) + \lambda_h h_y H_s/(h_s H_y) + \lambda_c c_y C_s/(c_s C_y) \; ,$$
$$v = \lambda_b B_y b_s/(B_s b_y) + \lambda_h H_y h_s/(H_s h_y) + \lambda_c C_y c_s/(C_s c_y) \; ,$$
$$T = \lambda_b b_s B_s/(b_y B_y) + \lambda_h h_s H_s/(h_y H_y) + \lambda_c c_s C_s/(c_y C_y) \; ,$$
$$R = b_s B_s/(b_y B_y) + h_s H_s/(h_y H_y) + c_s C_s/(c_y C_y) \; ,$$
$$d = b_s/(b_y \lambda_b) + h_s/(h_y \lambda_h) + c_s/(c_y \lambda_c) \; ,$$
$$D = B_s/(B_y \lambda_b) + H_s/(H_y \lambda_h) + C_s/(C_y \lambda_c) \; ,$$
$$1/\Lambda = 1/\lambda_b + 1/\lambda_h + 1/\lambda_c \; , X = \lambda_b \lambda_h \lambda_c \; . \qquad (85)$$

The formula for the amplitudes of three reggeon exchanges reads

$$A_j^{(3)} = \frac{C_{sh}^{(3)}}{3!} \sum_{b,c,h} f_I(b,c,h) \eta_b(0) \eta_h(0) \eta_c(0) (s/s_0)^{\alpha_b(0)+\alpha_h(0)+\alpha_c(0)-3}$$
$$\exp\{-\Lambda \Delta_T^2\} a_j^{(3)}(b,h,c) \; . \qquad (86)$$

When all three reggeons have isospin $T = 0$, then the isospin factor $f_I(b,h,c) = 1$ and the nonzero amplitudes $a_j^{(3)}(b,h,c)$ are

$$a_1^{(3)}(b,h,c) = -\frac{\Lambda}{X}\{S + \frac{\Lambda}{X}(1-\Lambda\Delta_T^2)(Y_s e + S_y E)$$
$$+\frac{Y\Lambda}{2X}[T\Delta_T^2 + (1-\Lambda\Delta_T^2)(2R - cC) + \frac{2\Lambda}{X}(2 - 4\Lambda\Delta_T^2 + \Lambda^2\Delta_T^4)eE]\} \; ,$$
$$a_2^{(3)}(b,h,c) = -i\Delta_T \frac{\Lambda^2}{X}\{Sw + \frac{\Lambda}{X}(2-\Lambda\Delta_T^2)(Y_s + S_y E w) + \frac{S_y}{2X}(V - aE)$$
$$+\frac{Y}{3X}[-(5/4 - \Lambda\Delta_T^2)C + \Lambda(2-\Lambda\Delta_T^2)D + \frac{2\Lambda^2}{X}(6 - 6\Lambda\Delta_T^2 + \Lambda^2\Delta_T^4)E]\} \; ,$$
$$a_6^{(3)}(b,h,c) = -i\Delta_T \frac{\Lambda^2}{X}\{SW + \frac{\Lambda}{X}(2-\Lambda\Delta_T^2)(S_y + Y_s e W) + \frac{Y_s}{2X}(v - Ae)$$
$$+\frac{Y}{3X}[-(5/4 - \Lambda\Delta_T^2)c + \Lambda(2-\Lambda\Delta_T^2)d + \frac{2\Lambda^2}{X}(6 - 6\Lambda\Delta_T^2 + \Lambda^2\Delta_T^4)e]\} \; ,$$
$$a_3^{(3)}(b,h,c) = -\frac{\Lambda}{2X}[-SU + S\Lambda w W(1 - 2\Lambda\Delta_T^2) + \frac{\Lambda}{3X}(-2 + 3\Lambda\Delta_T^2)(Y_s A + S_y a)$$
$$+\frac{\Lambda^2}{X}(2 - 7\Lambda\Delta_T^2 + 2\Lambda^2\Delta_T^4)(Y_s W + S_y w)] - \frac{Y\Lambda}{36X^2}[-2 - 3\Lambda\Delta_T^2 - 2\Lambda^2\Delta_T^4$$
$$\frac{2\Lambda^2}{X}(\lambda_b + \lambda_h + \lambda_c)(-8 + 25\Lambda\Delta_T^2 - 7\Lambda^2\Delta_T^4) + \Lambda^2(\lambda_b^{-2} + \lambda_h^{-2} + \lambda_c^{-2})$$
$$(2 - 7\Lambda\Delta_T^2 + 2\Lambda^2\Delta_T^4) + 18\frac{\Lambda^3}{X}(6 - 30\Lambda\Delta_T^2 + 17\Lambda^2\Delta_T^4 - 2\Lambda^3\Delta_T^6)] \; ,$$
$$a_4^{(3)}(b,h,c) = -\frac{\Lambda}{2X}\{-SU + S\Lambda w W + \frac{\Lambda}{3X}(2 - \Lambda\Delta_T^2)$$
$$[3\Lambda(Y_s W + S_y w) - Y_s A - S_y a]$$

$$+\frac{Y}{18X}[-2-\Lambda\Delta_T^2-\frac{2\Lambda^2}{X}(\lambda_b+\lambda_h+\lambda_c)(8-7\Lambda\Delta_T^2+\Lambda^2\Delta_T^4)$$
$$+\Lambda^2(\lambda_b^{-2}+\lambda_h^{-2}+\lambda_c^{-2})(2-\Lambda\Delta_T^2)+18\frac{\Lambda^3}{X}(6-6\Lambda\Delta_T^2+\Lambda^2\Delta_T^4)]\}\ . \quad (87)$$

When only one reggeon has isospin $T=1$, then the isospin factor $f_I(b,c,h)$ in (87) is to be put equal to expression (80) or (81). When two reggeons (say b and h) have $T=1$, then $f_I(b,c,h)$ is given by (82) and the new nonzero amplitude $a_5^{(3)}(b,h,c)$ appears. The formula for it reads

$$a_5^{(3)}(b,h,c)=-\frac{Y\Lambda^2}{27X^2}\{T\Delta_T^2+[2(R+\frac{b_sH_s}{b_yH_y}+\frac{B_sh_s}{B_yh_y})-cC](1-\Lambda\Delta_T^2)(\vec{\tau}_1\cdot\vec{\tau}_j)\ . \quad (88)$$

It has been already pointed out that we can substitute $2m_j$ instead of $(\vec{\tau}_1\cdot\vec{\tau}_j)$ in (88).

Non-spectator graphs

The contributions of the non-spectator graphs are given by (44) and (45) the latter formula describing charge exchange in which the $u\bar{u}$-pair is transformed into the $d\bar{d}$-pair and vice versa. We start with discussion of the contributions given by (44). In the first two terms in the brackets in (44) we are to rest two and three reggeon exchanges only not to go beyond our approximation. As we do not consider in the present paper nucleon charge exchange we can make use of the substitutions $(\vec{\tau}_1\cdot\vec{\tau}_2)\to 2m_q$ in the amplitude of quark-nucleon elastic scattering A_{m_q} and $(\vec{\tau}_1\cdot\vec{\tau}_3)\to -2m_q$ in the antiquark-nucleon scattering amplitude B_{-m_q} since $m_{\bar{q}}=-m_q$. The third term in the brackets in (44) is nonzero for three reggeon exchanges only. It can be represented by the sum of two terms

$$\hat{D}(\vec{\Delta}_1,\vec{\Delta}_2)=\hat{D}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)+\hat{D}_{1+2}(\vec{\Delta}_1,\vec{\Delta}_2)\ . \quad (89)$$

In (89) $\hat{D}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ corresponds to the graphs with exchanges of two reggeons b and h between the quark and the proton (the total momentum transferred to the nucleon is equal to Δ_1), the antiquark and the proton interacting through exchange with a reggeon c having the momentum Δ_2. The term $\hat{D}_{1+2}(\vec{\Delta}_1,\vec{\Delta}_2)$ describes exchanges with one reggeon c having the momentum Δ_1 between the quark and the proton and with two reggeons $b,\ h$ between the antiquark and proton (with the total momentum of the $b+h$ system equal to Δ_2). The formula for $\hat{D}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ reads

$$\begin{aligned}\hat{D}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)&=D_1(\vec{\sigma}_1\cdot\vec{l}\times\vec{\Delta}_2)+D_2(\vec{\sigma}_1\cdot\vec{l}\times\vec{\Delta}_1)+D_3(\vec{\sigma}_1\cdot\vec{l}\times\vec{\Delta}_2)(\vec{\sigma}_2\cdot\vec{l}\times\vec{\Delta}_1)\\
&+D_4(\vec{\sigma}_1\cdot\vec{l}\times\vec{\Delta}_1)(\vec{\sigma}_2\cdot\vec{l}\times\vec{\Delta}_2)+D_5(\vec{\sigma}_1\cdot\vec{l}\times\vec{\Delta}_1)(\vec{\sigma}_2\cdot\vec{l}\times\vec{\Delta}_1)+D_6(\vec{\sigma}_{1T}\cdot\vec{\sigma}_{2T})\\
&+D_7(\vec{\sigma}_1\cdot\vec{l})(\vec{\sigma}_2\cdot\vec{l})+D_8(\vec{\sigma}_1\cdot\vec{l}\times\vec{\Delta}_2)(\vec{\sigma}_3\cdot\vec{l}\times\vec{\Delta}_2)+D_9(\vec{\sigma}_1\cdot\vec{l}\times\vec{\Delta}_1)(\vec{\sigma}_3\cdot\vec{l}\times\vec{\Delta}_2)\\
&+D_{10}(\vec{\sigma}_1\cdot\vec{l}\times\vec{\Delta}_2)(\vec{\sigma}_2\cdot\vec{l}\times\vec{\Delta}_1)(\vec{\sigma}_3\cdot\vec{l}\times\vec{\Delta}_2)\\
&+D_{11}(\vec{\sigma}_1\cdot\vec{l}\times\vec{\Delta}_1)(\vec{\sigma}_2\cdot\vec{l}\times\vec{\Delta}_2)(\vec{\sigma}_3\cdot\vec{l}\times\vec{\Delta}_2)\\
&+D_{12}(\vec{\sigma}_1\cdot\vec{l}\times\vec{\Delta}_1)(\vec{\sigma}_2\cdot\vec{l}\times\vec{\Delta}_1)(\vec{\sigma}_3\cdot\vec{l}\times\vec{\Delta}_2)+D_{13}(\vec{\sigma}_{1T}\cdot\vec{\sigma}_{2T})(\vec{\sigma}_3\cdot\vec{l}\times\vec{\Delta}_2)\\
&+D_{14}(\vec{\sigma}_1\cdot\vec{l})(\vec{\sigma}_2\cdot\vec{l})(\vec{\sigma}_3\cdot\vec{l}\times\vec{\Delta}_2)\end{aligned} \quad (90)$$

where $(\vec{\sigma}_{1T} \cdot \vec{\sigma}_{2T})$ has been defined after relation (23). Every amplitude D_j in (90) is a sum over contributions of the reggeons $b,\ c,\ h$

$$D_j = \sum_{b,h,c} f_I(b,h,c) D_j^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) \ , \tag{91}$$

the reggeon c being exchanged between the proton and the antiquark, the reggeons $b,\ h$ being emitted by the quark. When all exchanged reggeons have isospins equal to zero, then D_j for $j \le 7$ are given by

$$\begin{aligned}
D_1^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) &= id[s_2(2\lambda\Delta_1^2 - 1) - y\Delta_1^2 - \frac{y\lambda}{x}(2 - 7\lambda\Delta_1^2 + 2\lambda^2\Delta_1^4)] \ , \\
D_2^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) &= id(\vec{\Delta}_1 \cdot \vec{\Delta}_2)\{-2s_2\lambda + y[1 - \frac{2\lambda^2}{x}(3 - \lambda\Delta_1^2)\} \ , \\
D_3^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) &= -ds_2[O + O_y(2\lambda\Delta_1^2 - 3)] \ , \\
D_4^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) &= -ds_2[O_y - O/2] \ , \\
D_5^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) &= 2ds_2\lambda(\vec{\Delta}_1 \cdot \vec{\Delta}_2)O_y \ , \\
D_6^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) &= -ds_2(\vec{\Delta}_1 \cdot \vec{\Delta}_2)[O_y - O/2] \ , \\
D_7^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) &= 0
\end{aligned} \tag{92}$$

with the isospin factor in (91) $f_I(b,h,c) = 1$ for this case. We have applied the short notations in (92)

$$\begin{aligned}
d = C_{sh}^{(3)} \frac{i\eta_b(0)\eta_h(0)\eta_c(0)}{3(\lambda_b + \lambda_h)^2} (zs/s_0)^{\alpha_b(0)+\alpha_h(0)-2} [(1-z)s/s_0]^{\alpha_c(0)-1} \\
B_y H_y C_y \tilde{c}_s \exp\{-\lambda\Delta_1^2 - \lambda_c\Delta_2^2\} \ , \\
\lambda = \lambda_b\lambda_h/(\lambda_b + \lambda_h) \ , \\
s_2 = b_s h_s \ , \\
y = b_y h_y \ , \\
x = \lambda_b\lambda_h \ , \\
O = b_y/b_s + h_y/h_s \ , \\
O_y = \lambda[b_y/(b_s\lambda_b) + h_y/(h_s\lambda_h)] \ .
\end{aligned} \tag{93}$$

For $8 \le l \le 14$ D_l are given by the relation

$$D_{j+7}^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) = iD_j^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2)\tilde{c}_y/\tilde{c}_s \tag{94}$$

with $\bar{q}\bar{q}a$ vertices $\tilde{a}_s$, $\tilde{a}_y$ being equal to $(a = b,\ h,\ c)$

$$\tilde{a}_s = (-1)^{T_a}\sigma_a a_s \ , \quad \tilde{a}_y = (-1)^{T_a}\sigma_a a_y \ . \tag{95}$$

We would like to stress that the longitudinal components $D_7 = D_{14} = 0$ in (90) when $T_b = T_h = T_c = 0$. Formulæ for $\hat{D}_{1+2}(\vec{\Delta}_1, \vec{\Delta}_2)$ can be obtained from (90), (91), (92), (93), (94) through transformations

$$b_{s,y} \to \tilde{b}_{s,y} \ , h_{s,y} \to \tilde{h}_{s,y} \ , \tilde{c}_{s,y} \to c_{s,y} \ , \Delta_1 \leftrightarrow \Delta_2 \ , \vec{\sigma}_2 \leftrightarrow \vec{\sigma}_3 \ , \vec{\tau}_2 \leftrightarrow \vec{\tau}_3 \tag{96}$$

(see definition of $\tilde{a}_s,\ \tilde{a}_y$ in (95)).

When only one reggeon has nonzero isospin, then we have for the case of $\hat{D}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ $f_I(b,h,c)=2m_q$ in (91) if $T_c=0$. For $T_c=1$ $f_I(b,h,c)=2m_{\bar{q}}=-2m_q$ hence for these two cases we can write $f_I(b,h,c)=2m_q(-1)^{T_c}$. Vice versa $\hat{D}_{1+2}(\vec{\Delta}_1,\vec{\Delta}_2)$ contains the factor $(-2m_q)$ when $T_c=0$ and $2m_q$ for $T_c=1$ hence $f_I(b,h,c)=-2m_q(-1)^{T_c}$. Formulæ (92) for D_j for all the cases discussed above remain valid. When the reggeon c and one reggeon among b and h have isospins equal to 1 we have $f_I(b,h,c)=-1$ both for $\hat{D}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ and $\hat{D}_{1+2}(\vec{\Delta}_1,\vec{\Delta}_2)$. When $T_b=T_h=1$ a reggeon c is the pomeron. In this case $f_I(b,h,c)=3$ but D_7 and D_{14} become nonzero. The formula for D_7 looks like

$$D_7=\frac{1}{3}yd(\vec{\tau}_1\cdot\vec{\tau}_j)(\vec{\Delta}_1\cdot\vec{\Delta}_2)(B_s/B_y+H_s/H_y)\ , \tag{97}$$

where $j=2$ in (97) for $\hat{D}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ and $j=3$ for $\hat{D}_{1+2}(\vec{\Delta}_1,\vec{\Delta}_2)$. The quantity of D_{14} for $\hat{D}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ is given by (94). The expression for $\hat{D}_{1+2}(\vec{\Delta}_1,\vec{\Delta}_2)$ can be obtained from the formula for $\hat{D}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ through transformations (96) as for the case when $T_b=T_h=T_c=0$.

The contribution of charge exchange is described by formula (45). In the first and second terms in the brackets in (45) we are to rest the two and three reggeon exchange contributions only. In the former case both reggeons emitted with the quark and the antiquark have isospins equal to 1. Their electric charges are of opposite signs to conserve the electric charge of the proton. For three reggeon exchanges we have additional pomeron emission either with the quark or with the antiquark. The third term in the brackets in (45) is absent for two reggeon exchanges. For the three reggeon exchange contribution it can be represented as the sum of two terms

$$\hat{E}(\vec{\Delta}_1,\vec{\Delta}_2)=\hat{E}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)+\hat{E}_{1+2}(\vec{\Delta}_1,\vec{\Delta}_2)\ . \tag{98}$$

The meaning of the terms $\hat{E}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ and $\hat{E}_{1+2}(\vec{\Delta}_1,\vec{\Delta}_2)$ in (98) is analogous to the meaning of $\hat{D}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ and $\hat{D}_{1+2}(\vec{\Delta}_1,\vec{\Delta}_2)$ in (89). The amplitude $\hat{E}_{1+2}(\vec{\Delta}_1,\vec{\Delta}_2)$ can be obtained from $\hat{E}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ via transformations (96). The isospin structure of $\hat{E}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ is given by the relation

$$\hat{E}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)=\hat{\Theta}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)(\vec{\tau}_2\cdot\vec{\tau}_3)+i\hat{\Phi}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)(\vec{\tau}_1\cdot\vec{\tau}_2\times\vec{\tau}_3)\ . \tag{99}$$

The spin structure of $\hat{\Theta}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ is the same as $\hat{D}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ which is given by (90). More over formulæ (90), (91), (92) after replacements $\hat{\Theta}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)\rightarrow\hat{D}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$, $D_j\rightarrow\Phi_j$, $D_j^{(b,h,c)}(\vec{\Delta}_1,\vec{\Delta}_2)\rightarrow\Phi_j^{(b,h,c)}(\vec{\Delta}_1,\vec{\Delta}_2)$ remain true in which we are to put $f_I(b,h,c)=1$. But we should rest now in the sum in (91) only such sets of $b,\ h,\ c$ when one reggeon among $b,\ h$ is the pomeron and other two reggeons in a set have isospins equal to 1. The spin structure of the second term in (99) $\hat{\Phi}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)$ looks like

$$\begin{aligned}\hat{\Phi}_{2+1}(\vec{\Delta}_1,\vec{\Delta}_2)&=\Phi_1(\vec{\sigma}_1\cdot\vec{l})+\Phi_2(\vec{\sigma}_1\cdot\vec{l})(\vec{\sigma}_2\cdot\vec{l}\times\vec{\Delta}_1)+\Phi_3(\vec{\sigma}_1\cdot\vec{l})(\vec{\sigma}_2\cdot\vec{\Delta}_2)\\+\Phi_4(\vec{\sigma}_1\cdot\vec{l})(\vec{\sigma}_2\cdot\vec{\Delta}_1)&+\Phi_5(\vec{\sigma}_1\cdot\vec{l})(\vec{\sigma}_3\cdot\vec{l}\times\vec{\Delta}_2)+\Phi_6(\vec{\sigma}_1\cdot\vec{l})(\vec{\sigma}_2\cdot\vec{l}\times\vec{\Delta}_1)(\vec{\sigma}_3\cdot\vec{l}\times\vec{\Delta}_2)\\&+\Phi_7(\vec{\sigma}_1\cdot\vec{l})(\vec{\sigma}_2\cdot\vec{\Delta}_2)(\vec{\sigma}_3\cdot\vec{l}\times\vec{\Delta}_2)+\Phi_8(\vec{\sigma}_1\cdot\vec{l})(\vec{\sigma}_2\cdot\vec{\Delta}_1)(\vec{\sigma}_3\cdot\vec{l}\times\vec{\Delta}_2)\ ,\end{aligned}$$

where Φ_j for $j \leq 4$ are

$$\Phi_j = \sum_{b,h,c} \Phi_j^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) ,$$

$$\Phi_1^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) = 2i\kappa \frac{B_s}{B_y}(\vec{l} \cdot \vec{\Delta}_1 \times \vec{\Delta}_2)\{2s_2\lambda_b - y[1 - 2\frac{\lambda}{\lambda_p}(2 - \lambda\Delta_1^2)]\} ,$$

$$\Phi_2^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) = -4\kappa \frac{y\lambda}{\lambda_p}\frac{B_s}{B_y}(\vec{l} \cdot \vec{\Delta}_1 \times \vec{\Delta}_2)(\lambda_p p_s/p_y + \lambda_b b_s/b_y) ,$$

$$\Phi_3^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) = 2\kappa y \frac{B_s}{B_y}(p_s/p_y + b_s/b_y) ,$$

$$\Phi_4^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) = \kappa y \frac{C_s}{C_y}(p_s/p_y + b_s/b_y) . \qquad (100)$$

Formulæ (100) have been written for the case when the reggeon h is the pomeron and reggeons b and c have isospins $T = 1$. When $b = P$ and $T_h = T_c = 1$ we are to replace in (100) $B_s,\ B_y,\ b_s,\ b_y$ with $H_s,\ H_y,\ h_s,\ h_y$, respectively. For $\Phi_k^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2)$ with $k \geq 5$ we are to use the relations

$$\Phi_{j+4}^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2) = i\Phi_j^{(b,h,c)}(\vec{\Delta}_1, \vec{\Delta}_2)\tilde{c}_y/\tilde{c}_s \qquad (101)$$

where $j \leq 4$. The quantity κ is given by the formula

$$\kappa = iC_{sh}^{(3)} \frac{\eta_b(0)\eta_p(0)\eta_c(0)}{6x^2}(zs/s_0)^{\alpha_b(0)+\alpha_h(0)-2}[(1-z)s/s_0]^{\alpha_c(0)-1}\lambda^2\tilde{c}_s$$
$$B_y P_y C_y \exp\{-\lambda\Delta_1^2 - \lambda_c\Delta_2^2\}$$

and the short notations $s_2,\ y,\ \lambda,\ x$ have been defined in (93).

Contributions of π-reggeon

The amplitude of quark/antiquark scattering on the proton due to one π-reggeon exchange is given by (17) and (73). To get the amplitudes of πP- and πPP-exchanges we have applied the representation

$$A_4^{(\pi)}(\vec{\sigma}_1 \cdot \vec{m})(\vec{\sigma}_j \cdot \vec{m}) = D^{(\pi)}(\vec{\sigma}_1 \cdot \vec{\Delta}_T)(\vec{\sigma}_j \cdot \vec{\Delta}_T)e^{-\lambda_\pi \Delta_T^2}\int_0^\infty \exp\{-\alpha(m_\pi^2 + \vec{\Delta}_T^2)\}d\alpha \qquad (102)$$

for the pole amplitude instead of (73). In (102) $j = 2$ ($j = 3$) for qN ($\bar{q}N$) scattering and

$$\lambda_\pi = \frac{r_\pi^2}{2} + \alpha'_\pi(m_\pi^2)\Big[\ln\Big(\frac{s}{s_0}\Big) - i\frac{\pi}{2}\Big] , \qquad (103)$$

$$D^{(\pi)} = \frac{3}{5}\frac{G_{\pi NN}^2}{16\pi m_N E_0}\frac{s_0 e^{-\lambda_\pi m_\pi^2}}{s - m_N^2 - m_q^2}(\vec{\tau}_1 \cdot \vec{\tau}_j) . \qquad (104)$$

Exponential representation (102) is convenient for calculations of integrals over transverse momenta $\vec{\Delta}_1,\ \vec{\Delta}_2,\ ...,\ \vec{\Delta}_n$ in formulæ (35). Putting (102) into (35) one can easily

get the expression for the total amplitude of $qN/\bar{q}N$ scattering due to πP-exchanges

$$
\begin{aligned}
A^{(\pi P)}(\vec{\Delta}_T) = \frac{1}{2}C_{sh}^{(2)}D^{(\pi)}\eta_P(0)\Big(\frac{s}{s_0}\Big)^{\alpha_P(0)-1} \\
\{ip_sP_s[J_2(\vec{\sigma}_{1T}\cdot\vec{\sigma}_{jT}) + 2\lambda_P^2 J_3(\vec{\sigma}_1\cdot\vec{\Delta}_T)(\vec{\sigma}_j\cdot\vec{\Delta}_T)] \\
-J_2[p_yP_s(\vec{\sigma}_1\cdot\vec{l}\times\vec{\Delta}_T) + p_sP_y(\vec{\sigma}_j\cdot\vec{l}\times\vec{\Delta}_T)] - i\Delta_T^2 p_yP_yJ_2\}\ ,
\end{aligned}
\tag{105}
$$

where we denote by J_n $(n=2,\ 3)$ the integrals

$$
J_n = (m_\pi)^{n-1}\int_0^1 \exp\{-\nu\Delta_T^2\}\frac{d\xi}{\varrho^n}
\tag{106}
$$

with ϱ and ν given by

$$
\begin{aligned}
\varrho &= (\lambda_P+\lambda_\pi)m_\pi^2 - \ln\xi\ , \\
\nu &= \lambda_P(\lambda_\pi m_\pi^2 - \ln\xi)/\varrho\ .
\end{aligned}
\tag{107}
$$

Comparing (105) with (17) one can easily get from $A^{(\pi P)}(\vec{\Delta}_T)$ the invariant amplitudes $A_j^{(\pi P)}(\vec{\Delta}_T)$ $(j=1,\ 2,...,\ 6)$.

For the πPP-exchanges we can get with the aid of (102) and (35) the invariant amplitudes $A_j^{(\pi PP)}(\vec{\Delta}_T)$

$$
\begin{aligned}
A_1^{(\pi PP)}(\vec{\Delta}_T) &= -\frac{\chi^{(\pi)}p_yp_sP_yP_s\Delta_T^2}{\lambda_P}\int_0^1 e^{-\Upsilon\Delta_T^2}\frac{d\xi}{\beta^2}\ , \\
A_2^{(\pi PP)}(\vec{\Delta}_T) &= i\Delta_T\chi^{(\pi)}\Big\{\frac{p_s^2P_sP_y}{\lambda_P}\int_0^1 e^{-\Upsilon\Delta_T^2}\frac{d\xi}{\beta^2} + \frac{p_y^2P_sP_y}{3\lambda_P^2}\int_0^1 e^{-\Upsilon\Delta_T^2}(\lambda_\pi+\alpha) \\
&\qquad \Big[2-\lambda_P\Delta_T^2(1+\Upsilon/\lambda_P)\Big]\frac{d\xi}{\beta^3}\Big\}\ , \\
A_3^{(\pi PP)}(\vec{\Delta}_T) &= -\frac{\Delta_T^2\chi^{(\pi)}}{18\lambda_P^2}\int_0^1 e^{-\Upsilon\Delta_T^2}(\lambda_\pi+\alpha)\Big\{6\lambda_P(p_y^2P_s^2+p_s^2P_y^2) \\
&+4p_y^2P_y^2\Big[1-\Upsilon\Delta_T^2-\frac{\lambda_P}{4(\lambda_\pi+\alpha)}-\frac{\Upsilon}{2\beta}(12-8\Upsilon\Delta_T^2+\Upsilon^2\Delta_T^4)\Big]\Big\}\frac{d\xi}{\beta^3}+\delta\ , \\
A_4^{(\pi PP)}(\vec{\Delta}_T) &= \Delta_T^2\chi^{(\pi)}\Big\{\Big(\lambda_Pp_s^2P_s^2+\frac{p_y^2P_y^2}{6\lambda_P}\Big)\int_0^1 e^{-\Upsilon\Delta_T^2}\frac{d\xi}{\beta^3} \\
&+\frac{1}{3}(p_y^2P_s^2+p_s^2P_y^2)\int_0^1 e^{-\Upsilon\Delta_T^2}(\lambda_\pi+\alpha)(3-\Upsilon\Delta_T^2)\frac{d\xi}{\beta^4}\Big\}+\delta\ , \\
A_5^{(\pi PP)}(\vec{\Delta}_T) &= 0\ , \\
A_6^{(\pi PP)}(\vec{\Delta}_T) &= i\Delta_T\chi^{(\pi)}\Big\{\frac{p_sp_yP_s^2}{\lambda_P}\int_0^1 e^{-\Upsilon\Delta_T^2}\frac{d\xi}{\beta^2} + \frac{p_sp_yP_y^2}{3\lambda_P^2}\int_0^1 e^{-\Upsilon\Delta_T^2}(\lambda_\pi+\alpha) \\
&\qquad \Big[2-\lambda_P\Delta_T^2(1+\Upsilon/\lambda_P)\Big]\frac{d\xi}{\beta^3}\Big\}\ ,
\end{aligned}
\tag{108}
$$

where α, β, Υ and $\chi^{(\pi)}$ in (108) denote

$$
\alpha = -\frac{1}{m_\pi^2}\ln\xi\ ,
$$

$$\beta = \lambda_P + 2(\lambda_\pi + \alpha)\ , \qquad (109)$$
$$\Upsilon = \lambda_P(\lambda_\pi + \alpha)/\beta\ ,$$
$$\chi^{(\pi)} = -\frac{C_{sh}^{(3)} D^{(\pi)} \eta_P^2(0)}{2! m_\pi^2}\Big(\frac{s}{s_0}\Big)^{2\alpha_P(0)-2}\ .$$

The amplitude δ in (108) looks like

$$\delta = \chi^{(\pi)}\Big\{\frac{p_s^2 P_s^2}{\lambda_P}\int_0^1 e^{-\Upsilon\Delta_T^2}\frac{d\xi}{\beta^2} + \frac{p_y^2 P_s^2 + p_s^2 P_y^2}{3\lambda_P^2}\int_0^1 e^{-\Upsilon\Delta_T^2}(\lambda_\pi+\alpha)(2-\Upsilon\Delta_T^2)\frac{d\xi}{\beta^3}$$
$$+\frac{p_y^2 P_y^2}{18\lambda_P^3}\int_0^1 e^{-\Upsilon\Delta_T^2}(\lambda_\pi+\alpha)$$
$$\Big[2 - 4\Upsilon\Delta_T^2 + \Upsilon^2\Delta_T^4 + 4\frac{\lambda_P}{\Upsilon} + \frac{\lambda_P^2\Delta_T^2}{\lambda_\pi+\alpha} - \frac{\Upsilon}{\lambda_\pi+\alpha}(6 - 6\Upsilon\Delta_T^2 + \Upsilon^2\Delta_T^4)\Big]\frac{d\xi}{\beta^3}\Big\}\ .$$

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